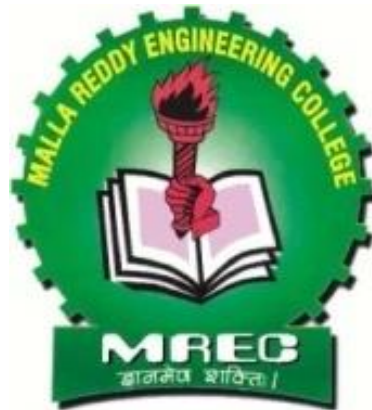


BASIC ELECTRICAL AND ELECTRONICS ENGINEERING LECTURE NOTES

(80201-MR18)

Prepared By:

P KAMALAKAR Associate Professor, EEE
V GANESH KUMAR Assistant Professor, EEE



DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

MALLA REDDY ENGINEERING COLLEGE AUTONOMOUS

MAISAMMAGUDA, DULAPALLY– 500014, Hyderabad

**MALLA REDDY ENGINEERING COLLEGE AUTONOMOUS
(AUTONOMOUS)**

MAISAMMAGUDA, DULAPALLY– 500014, Hyderabad

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

LECTURE NOTES:

Course Title	Basic Electrical and Electronics Engineering (ME/EEE/CE/CSE/ECE/IT/MINING)			
Course Code	80201			
Course Structure	Lectures	Tutorials	Practicals	Credits
	3	-	-	3
Course Coordinator	Mr. P KAMALAKAR, Assistant Professor,EEE Mr.V GANESH KUMAR Assistant Professor,EEE			

SYLLABUS:

Module I: DC Circuits

9 Periods

Electrical circuit elements (R, L and C), voltage and current sources, Kirchhoff's current and voltage laws - Series, parallel, series-parallel, star-to-delta and delta-to-star transformation- analysis of simple circuits with dc excitation. Superposition, Thevenin's and Maximum Power Transfer Theorems with DC excitation.

Module II: AC Circuits

9 Periods

Representation of sinusoidal waveforms, peak and rms values, phasor representation, real power, reactive power, apparent power, power factor. Analysis of single-phase ac circuits consisting of R, L, C, RL, RC, RLC combinations (series and parallel).

Module III: Introduction to Electrical Machines

10 Periods

A: DC Machines : Construction & Principle of Operation of DC Generators – E.M.F Equation. Principle of operation DC Motors – Back E.M.F. - Torque equation – Brake Test -Characteristics.

B: AC Machines: Construction and Principle of operation of Transformer- EMF Equation. Construction and Principle of Operation of 3 Phase Induction Motors - Brake test on 3-Phase Induction Motor – Applications.

Module IV: P-N Junction Diode

10 Periods

P-N Junction Diode: Diode equation, Energy Band diagram, Volt-Ampere characteristics, Temperature dependence, Ideal versus practical, Static and dynamic resistances, Equivalent circuit, Diffusion and Transition Capacitances. Zener diode operation, Zener diode as voltage regulator.

Rectifiers : P-N junction as a rectifier - Half Wave Rectifier, Ripple Factor - Full Wave Rectifier, Bridge Rectifier.

Filters : Filters – Inductor Filters, Capacitor Filters, L- section Filters, π - section Filters.

Module V: BJT and Junction Field Effect Transistor (JFET)

10 Periods

Bipolar Junction Transistor (BJT): Construction, Principle of Operation, Symbol, Amplifying Action, Common Emitter, Common Base and Common Collector configurations and Input-Output Characteristics, Comparison of CE, CB and CC configurations

Junction Field Effect Transistor and MOSFET: Construction, Principle of Operation, Symbol, Pinch-Off Voltage, Volt-Ampere Characteristic, Comparison of BJT and FET.

Text Books

1. M.Surya Kalavathi, Ramana Pilla, Ch. Srinivasa Rao, Gulinindala Suresh, “**Basic Electrical and Electronics Engineering**”, S.Chand and Company Limited, New Delhi, 1st Edition, 2017.
2. R.L.Boylestad and Louis Nashlesky, “**Electronic Devices & Circuit Theory**”, Pearson Education, 2007.

References

1. V.K. Mehtha and Rohit Mehta, “**Principles of Electrical Engineering and Electronics**”, S.Chand & Co., 2009.
2. Jacob Milliman, Christos C .Halkias, Satyabrata Jit (2011), “**Electronic Devices and Circuits**”, 3rd edition, Tata McGraw Hill, New Delhi.
3. Thomas L. Floyd and R. P. Jain, “**Digital Fundamentals**”, Pearson Education, 2009.
4. David A. Bell, “**Electronic Devices and Circuits**”, Oxford University Press, 2008.
5. Nagrath I.J. and D. P. Kothari, “**Basic Electrical Engineering**”, Tata McGraw Hill, 2001.
6. Mittle N., “**Basic Electrical Engineering**”, Tata McGraw Hill Education, New Delhi, 2nd Edition, 2005.

E - Resources

1. <https://www.electrical4u.com/ohms-law-equation-formula-and-limitation-of-ohms-law/>
2. <https://www.eeweb.com/passives>
3. <http://nptel.ac.in/courses/108108076/>
4. <http://nptel.ac.in/downloads/108105053/>

UNIT – I

DIRECT CURRENT CIRCUITS

1.1 INTRODUCTION

Given an electrical network, the network analysis involves various methods. The process of finding the network variables namely the voltage and currents in various parts of the circuit is known as network analysis. Before we carry out actual analysis it is very much essential to thoroughly understand the various terms associated with the network. In this chapter we shall begin with the definition and understanding in detail some of the commonly used terms. The basic laws such as Ohm's law, KCL and KVL, those can be used to analyse a given network Analysis becomes easier if we can simplify the given network. We will be discussing various techniques, which involve combining series and parallel connections of R, L and C elements.

1.2 SYSTEMS OF UNITS

As engineers, we deal with measurable quantities. Our measurement must be communicated in standard language that virtually all professionals can understand irrespective of the country. Such an international measurement language is the International System of Units (SI). In this system, there are six principal units from which the units of all other physical quantities can be derived.

Quantity	Basic Unit	Symbol
Length	Meter	M
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	Kelvin	K
Luminous intensity	candela	Cd

One great advantage of SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit.

Multiplier	Prefix	Symbol
10^{12}	Tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	K
10^{-3}	milli	m
10^{-6}	micro	
10^{-9}	nano	n
10^{-12}	pico	p

1.3 BASIC CONCEPTS AND DEFINITIONS

1.3.1 CHARGE

The most basic quantity in an electric circuit is the electric charge. We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). Charge, positive or negative, is denoted by the letter q or Q.

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge 'e' on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron and the neutron has no charge. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

1.3.2 CURRENT

Current can be defined as the motion of charge through a conducting material, measured in Ampere (A). Electric current, is denoted by the letter i or I .

The unit of current is the ampere abbreviated as (A) and corresponds to the quantity of total charge that passes through an arbitrary cross section of a conducting material per unit second.

Mathematically,

$$I = \frac{Q}{t} \text{ or } Q = It$$

Where Q is the symbol of charge measured in Coulombs (C), I is the current in amperes (A) and t is the time in second (s).

The current can also be defined as the rate of charge passing through a point in an electric circuit. Mathematically,

$$i = \frac{dq}{dt}$$

The charge transferred between time t_1 and t_2 is obtained as

$$q = \int_{t_1}^{t_2} i dt$$

A constant current (also known as a direct current or DC) is denoted by symbol I whereas a time-varying current (also known as alternating current or AC) is represented by the symbol i or $i(t)$. Figure 1.1 shows direct current and alternating current.

Current is always measured through a circuit element as shown in Fig. 1.1

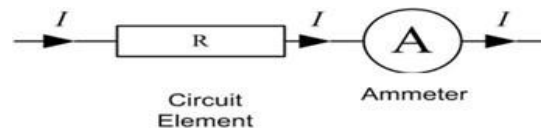


Fig. 1.1 Current through Resistor (R)

Two types of currents:

- 1) A direct current (DC) is a current that remains constant with time.
- 2) An alternating current (AC) is a current that varies with time.

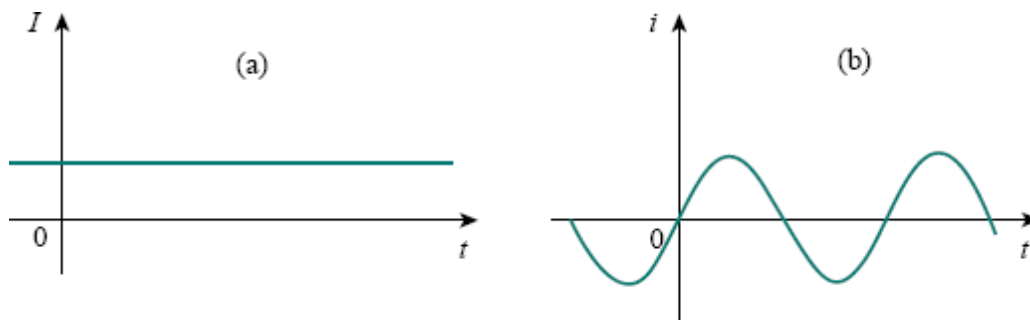


Fig. 1.2 Two common types of current: (a) direct current (DC), (b) alternative current (AC)

Example 1.1

Determine the current in a circuit if a charge of 80 coulombs passes a given point in 20 seconds (s).

Solution:

$$I = \frac{Q}{t} = \frac{80}{20} = 4 \text{ A}$$

Example 1.2

How much charge is represented by 4,600 electrons?

Solution:

Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have:

$$-1.602 \times 10^{-19} \times 4600 = -7.369 \times 10^{-16} \text{ C}$$

Example 1.3

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt} (5t \sin 4\pi t) = (5 \sin 4\pi t + 20\pi t \cos 4\pi t) \text{ mA}$$

At $t = 0.5$ s.

$$i = 31.42 \text{ mA}$$

Example 1.4

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Solution:

$$q = \int_{t=1}^{t=2} i dt = \int_1^2 (3t^2 - t) dt = \left(t^3 - \frac{t^2}{2} \right)_1^2 = (8 - 2) - \left(1 - \frac{1}{2} \right) = 5.5 \text{ C}$$

1.3.3 VOLTAGE (or) POTENTIAL DIFFERENCE

To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as voltage or potential difference. The voltage v_{ab} between two points a and b in an electric circuit is the energy (or work) needed to move a unit charge from a to b.

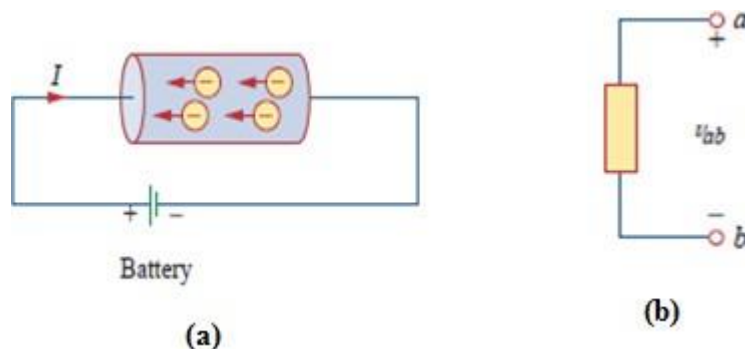


Fig. 1.3(a) Electric Current in a conductor, (b) Polarity of voltage v_{ab}

Voltage (or potential difference) is the energy required to move charge from one point to the other, measured in volts (V). Voltage is denoted by the letter v or V .

Mathematically,

$$v_{ab} = \frac{dw}{dq}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage v_{ab} or simply V is measured in volts (V).

$$1 \text{ volt} = 1 \text{ joule/coulomb} = 1 \text{ newton-meter/coulomb}$$

Fig. 1.3 shows the voltage across an element (represented by a rectangular block) connected to points a and b . The plus (+) and minus (-) signs are used to define reference direction or voltage polarity. The v_{ab} can be interpreted in two ways: (1) point a is at a potential of v_{ab} volts higher than point b , or (2) the potential at point a with respect to point b is v_{ab} . It follows logically that in general

$$v_{ab} = -v_{ba}$$

Voltage is always measured across a circuit element as shown in Fig. 1.4

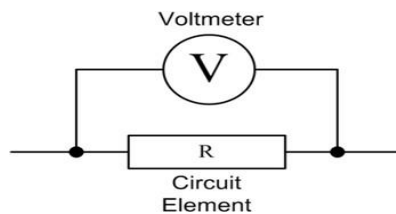


Fig. 1.4 Voltage across Resistor (R)

Example 1.5

An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:

$$\text{Total charge } dq = i \cdot dt = 2 \cdot 10 = 20$$

The voltage drop is

$$v = \frac{dw}{dq} = \frac{2.3 \cdot 10^3}{20} = 115 \text{ V}$$

1.3.4 POWER

Power is the time rate of expending or absorbing energy, measured in watts (W). Power, is denoted by the letter p or P .

Mathematically,

$$p = \frac{dw}{dt}$$

Where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s).

From voltage and current equations, it follows that;

$$p = \frac{dw}{dt} = \frac{dw}{dq} * \frac{dq}{dt} = V * I$$

Thus, if the magnitude of current I and voltage are given, then power can be evaluated as the product of the two quantities and is measured in watts (W).

Sign of power:

Plus sign: Power is absorbed by the element. (Resistor, Inductor)

Minus sign: Power is supplied by the element. (Battery, Generator)

Passive sign convention:

If the current enters through the positive polarity of the voltage, $p = +vi$

If the current enters through the negative polarity of the voltage, $p = -vi$

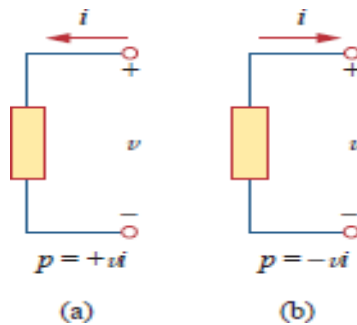


Fig 1.5 Polarities for Power using passive sign convention
(a) Absorbing Power (b) Supplying Power

1.3.5 ENERGY

Energy is the capacity to do work, and is measured in joules (J).

The energy absorbed or supplied by an element from time 0 to t is given by,

$$w = \int_0^t p dt = \int_0^t v i dt$$

The electric power utility companies measure energy in watt-hours (WH) or Kilo watt-hours (KWH)

$$1 \text{ WH} = 3600 \text{ J}$$

Example 1.6

A source e.m.f. of 5 V supplies a current of 3A for 10 minutes. How much energy is provided in this time?

Solution:

$$W = VIt = 5 \times 3 \times 10 \times 60 = 9 \text{ kJ}$$

Example 1.7

An electric heater consumes 1.8Mj when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.

Solution:

$$P = W / t = (1.8 \times 10^6) / (30 \times 60) = 1000$$

Power rating of heater = 1kW

$$P = VI$$

Thus

$$I = P/V = 1000/250 = 4$$

Hence the current taken from the supply is 4A.

1.4 OHM'S LAW

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

Ohm's law states that at constant temperature, the voltage (V) across a conducting material is directly proportional to the current (I) flowing through the material.

Mathematically,

$$\begin{aligned} V &\propto I \\ V &= RI \end{aligned}$$

Where the constant of proportionality R is called the resistance of the material. The V-I relation for resistor according to Ohm's law is depicted in Fig. 1.6

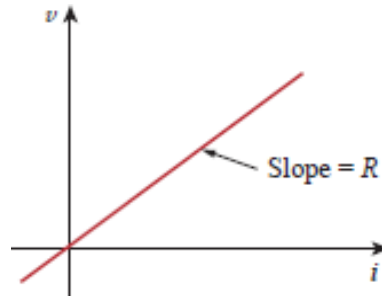


Fig. 1.6 V-I Characteristics for resistor

Limitations of Ohm's Law:

1. Ohm's law is not applicable to non-linear elements like diode, transistor etc.
2. Ohm's law is not applicable for non-metallic conductors like silicon carbide.

1.5 CIRCUIT ELEMENTS

An element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are 2 types of elements found in electrical circuits.

- a) Active elements (Energy sources):** The elements which are capable of generating or delivering the energy are called active elements.
E.g., Generators, Batteries
- b) Passive element (Loads):** The elements which are capable of receiving the energy are called passive elements.
E.g., Resistors, Capacitors and Inductors

1.5.1 ACTIVE ELEMENTS (ENERGY SOURCES)

The energy sources which are having the capacity of generating the energy are called active elements. The most important active elements are voltage or current sources that generally deliver power/energy to the circuit connected to them.

There are two kinds of sources

- a) Independent sources
- b) Dependent sources

1.5.1.1 INDEPENDENT SOURCES:

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

Ideal Independent Voltage Source:

An ideal independent voltage source is an active element that gives a constant voltage across its terminals irrespective of the current drawn through its terminals. In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. The symbol of ideal independent voltage source and its V-I characteristics are shown in Fig. 1.7

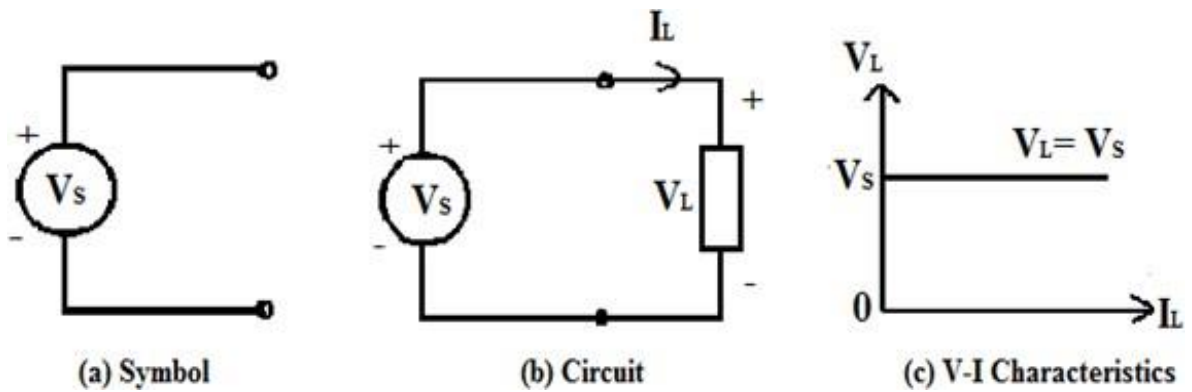


Fig. 1.7 Ideal Independent Voltage Source

Practical Independent Voltage Source:

Practically, every voltage source has some series resistance across its terminals known as internal resistance, and is represented by R_{se} . For ideal voltage source $R_{se} = 0$. But in practical voltage source R_{se} is not zero but may have small value. Because of this R_{se} voltage across the terminals decreases with increase in current as shown in Fig. 1.8

Terminal voltage of practical voltage source is given by

$$V_L = V_s - I_L R_{se}$$

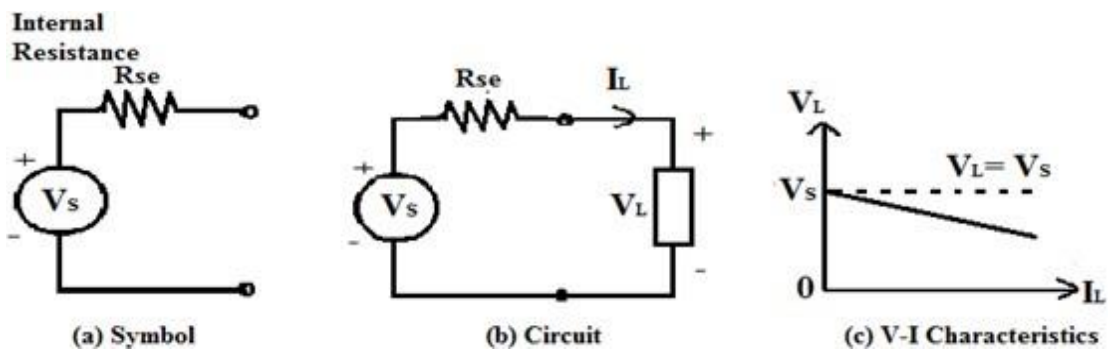


Fig. 1.8 Practical Independent Voltage Source

Ideal Independent Current Source:

An ideal independent Current source is an active element that gives a constant current through its terminals irrespective of the voltage appearing across its terminals. That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol of idea independent current source and its V-I characteristics are shown in Fig. 1.9

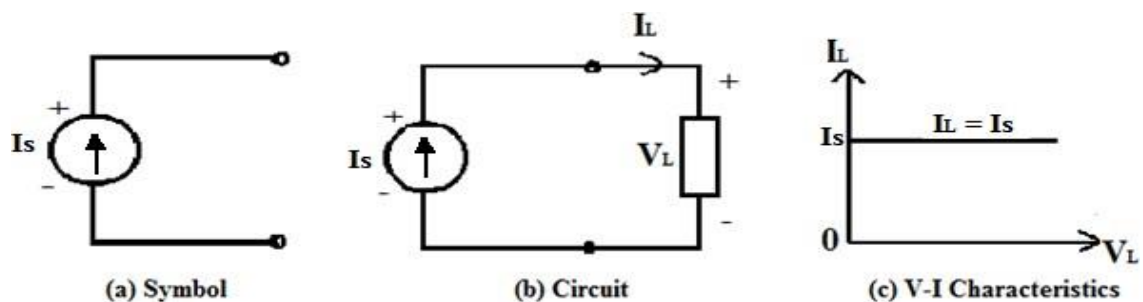


Fig. 1.9 Ideal Independent Current Source

Practical Independent Current Source:

Practically, every current source has some parallel/shunt resistance across its terminals known as internal resistance, and is represented by R_{sh} . For ideal current source $R_{sh} = \infty$ (infinity). But in practical voltage source R_{sh} is not infinity but may have a large value. Because of this R_{sh} current through the terminals slightly decreases with increase in voltage across its terminals as shown in Fig. 1.10.

Terminal current of practical current source is given by

$$I_L = I_s - I_{sh}$$

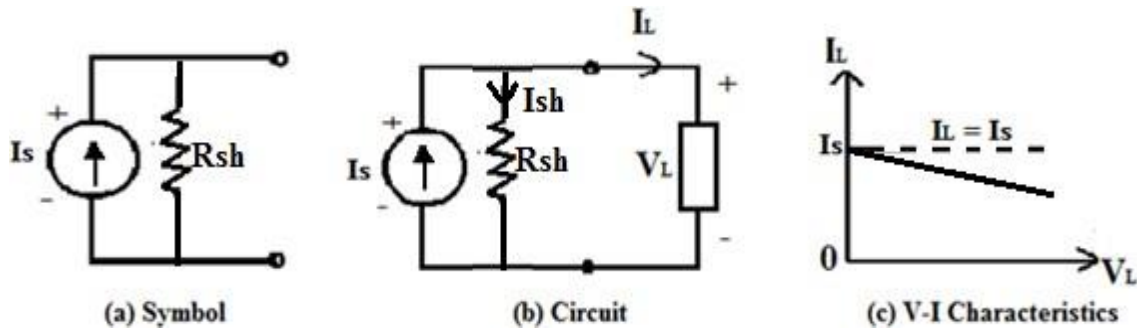


Fig. 1.10 Practical Independent Current Source

1.5.1.2 DEPENDENT (CONTROLLED) SOURCES

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.11. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS)
2. A current-controlled voltage source (CCVS)
3. A voltage-controlled current source (VCCS)
4. A current-controlled current source (CCCS)

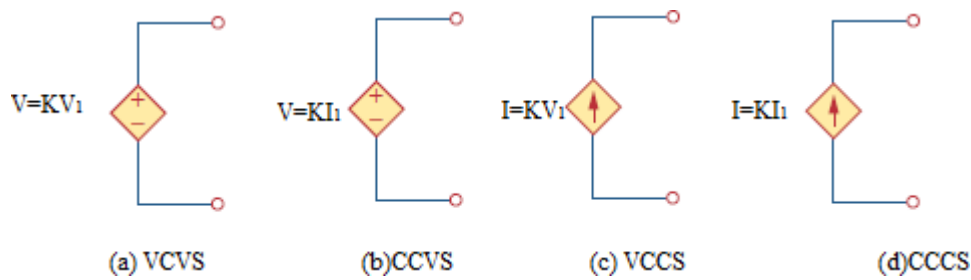


Fig. 1.11 Symbols for Dependent voltage source and Dependent current source

Dependent sources are useful in modeling elements such as transistors, operational amplifiers, and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.12, where the voltage $10i$ of the voltage source depends on the current i through element C. Students might be surprised that the value of the dependent voltage source is $10i$ V (and not $10i$ A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+ -) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

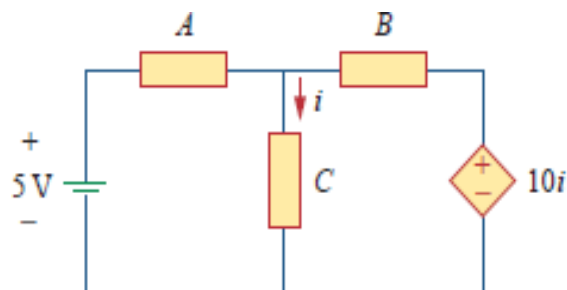


Fig. 1.12 The source in right hand side is current-controlled voltage source

1.5.2 PASSIVE ELEMENTS (LOADS)

Passive elements are those elements which are capable of receiving the energy. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element. More specifically, a passive element is defined as one that cannot supply average power that is greater than zero over an infinite time interval. Resistors, capacitors, Inductors fall in this category.

1.5.2.1 RESISTOR

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist the flow of current, is known as resistance and is represented by the symbol R . The Resistance is measured in ohms (Ω). The circuit element used to model the current-resisting behavior of a material is called the resistor.

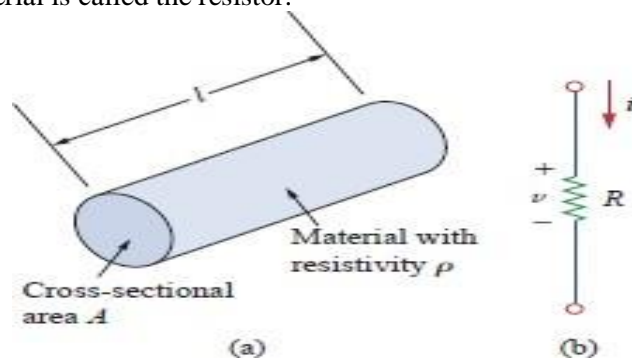


Fig. 1.13 (a) Typical Resistor, (b) Circuit Symbol for Resistor

The resistance of a resistor depends on the material of which the conductor is made and geometrical shape of the conductor. The resistance of a conductor is proportional to the its length (and inversely

proportional to its cross sectional area (A). Therefore the resistance of a conductor can be written as,

$$R = \frac{\rho l}{A}$$

The proportionality constant is called the specific resistance or resistivity of the conductor and its value depends on the material of which the conductor is made.

The inverse of the resistance is called the conductance and inverse of resistivity is called specific conductance or conductivity. The symbol used to represent the conductance is G and conductivity is σ . Thus conductivity and its units are Siemens per meter

$$G = \frac{1}{R} = \frac{A}{\rho l} = \frac{1}{\rho} \cdot \frac{A}{l} = \sigma \cdot \frac{A}{l}$$

By using Ohm's Law, The power dissipated in a resistor can be expressed in terms of R as below

$$P = VI = I^2R = \frac{V^2}{R}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$P = VI = V^2G = \frac{I^2}{G}$$

The energy lost in the resistor from time 0 to t is expressed as

$$W = \int_0^t P dt$$

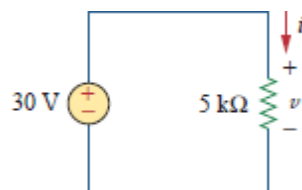
$$W = \int_0^t I^2 R dt = I^2 R t$$

$$W = \int_0^t \frac{V^2}{R} dt = \frac{V^2}{R} t$$

Where V is in volts, I is in amperes, R is in ohms, and energy W is in joules

Example 1.9

In the circuit shown in Fig. below, calculate the current i , the conductance G, the power p and energy lost in the resistor W in 2hours.



Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 6 \text{ mA}$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.2 \text{ mS}$$

We can calculate the power in various ways

$$p = vi = 30(6 \times 10^{-3}) = 180 \text{ mW}$$

or

$$p = i^2 R = (6 \times 10^{-3})^2 (5 \times 10^3) = 180 \text{ mW}$$

or

$$p = \frac{v^2}{R} = \frac{30^2}{5 \times 10^3} = 180 \text{ mW}$$

Energy lost in the resistor is

$$W = i^2 R t = (6 \times 10^{-3})^2 (5 \times 10^3) (2) = 360 \text{ mWhor} = 360 \text{ mJ}$$

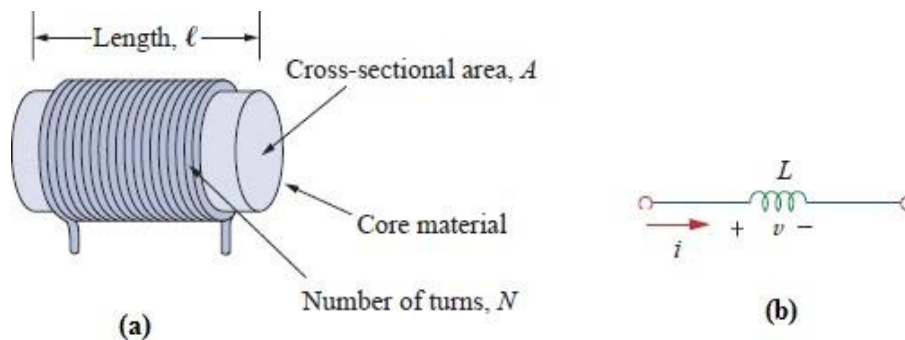
1.5.2.2 INDUCTOR

Fig. 1.14 (a) Typical Inductor, (b) Circuit symbol of Inductor

A wire of certain length, when twisted into a coil becomes a basic inductor. The symbol for inductor is shown in Fig.1.14 (b). If current is made to pass through an inductor, an electromagnetic field is formed. A change in the magnitude of the current changes the electromagnetic field. Increase in current expands the fields, and decrease in current reduces it. Therefore, a change in current produces change in the electromagnetic field, which induces a voltage across the coil according to Faraday's law of electromagnetic induction. i.e., the voltage across the inductor is directly proportional to the time rate of change of current.

Mathematically,

$$V \propto \frac{di}{dt}$$

$$v = L \frac{di}{dt}$$

Where L is the constant of proportionality called the inductance of an inductor. The unit of inductance is Henry (H).we can rewrite the above equation as

$$di = \frac{1}{L} v dt$$

Integrating both sides from time 0 to t, we get

$$\int_0^t di = \frac{1}{L} \int_0^t v dt$$

$$i(t) - i(0) = \frac{1}{L} \int_0^t v dt$$

$$i(t) = \frac{1}{L} \int_0^t v dt + i(0)$$

From the above equation we note that the current in an inductor is dependent upon the integral of the voltage across its terminal and the initial current in the coil $i(0)$.

The power absorbed by the inductor is

$$P = vi = Li \frac{di}{dt}$$

The energy stored by the inductor is

$$\begin{aligned} W &= \int_0^t P dt \\ &= \int_0^t Li \frac{di}{dt} dt = \frac{Li^2}{2} \end{aligned}$$

From the above discussion, we can conclude the following.

1. The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to DC.
2. A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor the current cannot change abruptly i.e., the inductor opposes the sudden changes in currents.
3. The inductor can store finite amount of energy. Even if the voltage across the inductor is zero
4. A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

1.5.2.2 CAPACITOR

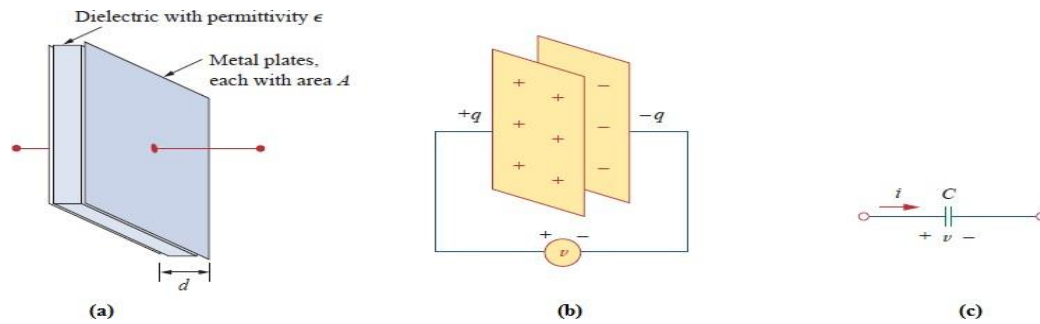


Fig. 1.15 (a) Typical Capacitor, (b) Capacitor connected to a voltage source, (c) Circuit Symbol of capacitor

Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor. The conducting surfaces are called electrodes, and the insulating medium is called dielectric. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges, and is concentrated within the dielectric.

When a voltage source v is connected to the capacitor, as in Fig 1.15 (c), the source deposits a positive charge q on one plate and a negative charge $-q$ on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q , is directly proportional to the applied voltage v so that

Where C , the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F).

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v , it does not depend on q or v . It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig.1.15 (a), the capacitance is given by

$$q = Cv$$

Where A is the surface area of each plate, d is the distance between the plates, and ϵ is the permittivity of the dielectric material between the plates.

The current flowing through the capacitor is given by

$$C = \frac{\epsilon A}{d}$$

$$i = \frac{dq}{dt}$$

$$i = C \frac{dv}{dt}$$

We can rewrite the above equation as

$$dv = \frac{1}{C} i dt$$

Integrating both sides from time 0 to t , we get

$$\int_0^t dv = \frac{1}{C} \int_0^t i dt$$

$$v(t) - v(0) = \frac{1}{C} \int_0^t i dt$$

$$v(t) = \frac{1}{C} \int_0^t i dt + v(0)$$

From the above equation we note that the voltage across the terminals of a capacitor is dependent upon the integral of the current through it and the initial voltage.

The power absorbed by the capacitor is

$$P = vi = vC \frac{dv}{dt}$$

The energy stored by the capacitor is

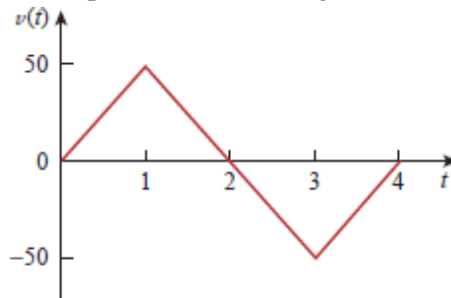
$$\begin{aligned} W &= \int_0^t P dt \\ &= \int_0^t vC \frac{dv}{dt} dt = \frac{Cv^2}{2} \end{aligned}$$

From the above discussion we can conclude the following,

1. The current in a capacitor is zero if the voltage across it is constant; that means, the capacitor acts as an open circuit to DC.
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly. i.e., A capacitor will oppose the sudden changes in voltages.
3. The capacitor can store a finite amount of energy, even if the current through it is zero.
4. A pure capacitor never dissipates energy, but only stores it; that is why it is called non-dissipative passive element. However, physical capacitors dissipate power due to internal resistance.

Example 1.10

Determine the current through a 200 capacitor whose voltage is shown in Fig. below



Solution:

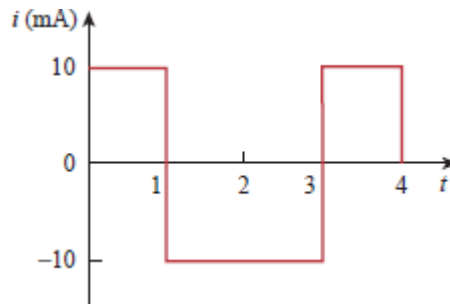
The voltage waveform can be described mathematically as

$$v(t) = \begin{cases} 50tV & 0 < t < 1 \\ 100 - 50tV & 1 < t < 3 \\ -200 + 50tV & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases}$$

Since $i = C \frac{dv}{dt}$ and $C = 200\mu F$, we take the derivative of $v(t)$ to obtain the $i(t)$

$$\begin{aligned} i(t) &= 200 \times 10^{-6} \times \begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases} \\ &= \begin{cases} 10 \text{ mA} & 0 < t < 1 \\ -10 \text{ mA} & 1 < t < 3 \\ 10 \text{ mA} & 3 < t < 4 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Hence, the current wave form is as shown in the fig. below



1.6 NETWORK/CIRCUIT TERMINOLOGY

In the following section various definitions and terminologies frequently used in electrical circuit analysis are outlined.

- **Network Elements:** The individual components such as a resistor, inductor, capacitor, diode, voltage source, current source etc. that are used in circuit are known as network elements.
- **Network:** The interconnection of network elements is called a network.
- **Circuit:** A network with at least one closed path is called a circuit. So, all the circuits are networks but all networks are not circuits.
- **Branch:** A branch is an element of a network having only two terminals.
- **Node:** A node is the point of connection between two or more branches. It is usually indicated by a dot in a circuit.
- **Loop:** A loop is any closed path in a circuit. A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- **Mesh or Independent Loop:** Mesh is a loop which does not contain any other loops in it.

1.7 KIRCHHOFF'S LAWS

The most common and useful set of laws for solving electric circuits are the Kirchhoff's voltage and current laws. Several other useful relationships can be derived based on these laws. These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

1.7.1 KIRCHHOFF'S CURRENT LAW (KCL)

This is also called as Kirchhoff's first law or Kirchhoff's nodal law. Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

Statement: Algebraic sum of the currents meeting at any junction or node is zero. The term 'algebraic' means the value of the quantity along with its sign, positive or negative.

Mathematically, KCL implies that

$$\sum_{n=1}^N i_n = 0$$

Where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

Alternate Statement: Sum of the currents flowing towards a junction is equal to the sum of the currents flowing away from the junction.

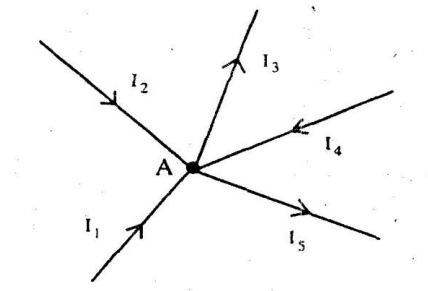


Fig 1.16 Currents meeting in a junction

Consider Fig. 1.16 where five branches of a circuit are connected together at the junction or node A. Currents I_1 , I_2 and I_4 are flowing towards the junction whereas currents I_3 and I_5 are flowing away from junction A. If a positive sign is assigned to the currents I_2 and I_4 that are flowing into the junction then the currents I_3 and I_5 flowing away from the junction should be assigned with the opposite sign i.e. the negative sign.

Applying Kirchhoff's current law to the junction A

$$I_1 + I_2 - I_3 + I_4 - I_5 = 0 \text{ (algebraic sum is zero)}$$

The above equation can be modified as $I_1 + I_2 + I_4 = I_3 + I_5$ (sum of currents towards the junction = sum of currents flowing away from the junction).

1.7.2 KIRCHHOFF'S VOLTAGE LAW (KVL)

This is also called as Kirchhoff's second law or Kirchhoff's loop or mesh law. Kirchhoff's second law is based on the principle of conservation of energy.

Statement: Algebraic sum of all the voltages around a closed path or closed loop at any instant is zero. Algebraic sum of the voltages means the magnitude and direction of the voltages; care should be taken in assigning proper signs or polarities for voltages in different sections of the circuit.

Mathematically, KVL implies that

$$\sum_{n=1}^N V_n = 0$$

Where N is the number of voltages in the loop (or the number of branches in the loop) and is the n^{th} voltage in a loop.

The polarity of the voltages across active elements is fixed on its terminals. The polarity of the voltage drop across the passive elements (Resistance in DC circuits) should be assigned with reference to the direction of the current through the elements with the concept that the current flows from a higher potential to lower potential. Hence, the entry point of the current through the passive elements should be marked as the positive polarity of voltage drop across the element and the exit point of the current as the negative polarity. The direction of currents in different branches of the circuits is initially marked either with the known direction or assumed direction.

After assigning the polarities for the voltage drops across the different passive elements, algebraic sum is accounted around a closed loop, either clockwise or anticlockwise, by assigning a particular sign, say the positive sign for all rising potentials along the path of tracing and the negative sign for all decreasing potentials. For example consider the circuit shown in Fig. 1.17

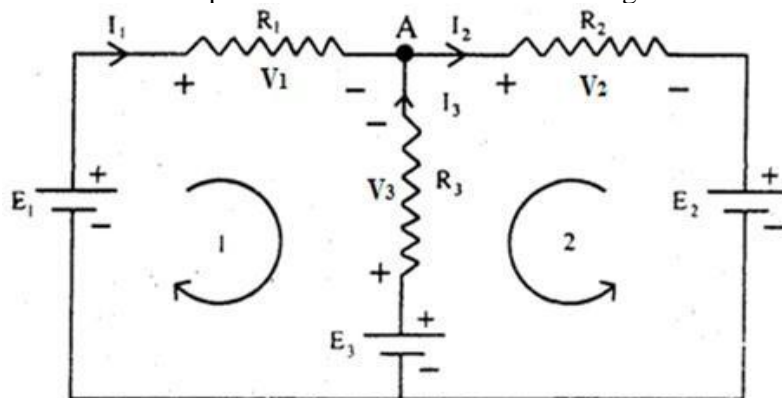


Fig. 1.17 Circuit for KVL

The circuit has three active elements with voltages E_1 , E_2 and E_3 . The polarity of each of them is fixed. R_1 , R_2 , R_3 are three passive elements present in the circuit. Currents I_1 and I_3 are marked flowing into the junction A and current I_2 marked away from the junction A with known information

or assumed directions. With reference to the direction of these currents, the polarity of voltage drops V_1 , V_2 and V_3 are marked.

For loop1 it is considered around clockwise

$$\begin{aligned} + E_1 - V_1 + V_3 - E_3 &= 0 \\ + E_1 - I_1 R_1 + I_3 R_3 - E_3 &= 0 \\ E_1 - E_3 &= I_1 R_1 - I_3 R_3 \end{aligned}$$

For loop2 it is considered anticlockwise

$$\begin{aligned} + E_2 + V_2 + V_3 - E_3 &= 0 \\ + E_2 + I_2 R_2 + I_3 R_3 - E_3 &= 0 \\ E_2 - E_3 &= -I_2 R_2 - I_3 R_3 \end{aligned}$$

Two equations are obtained following Kirchhoff's voltage law. The third equation can be written based on Kirchhoff's current law as

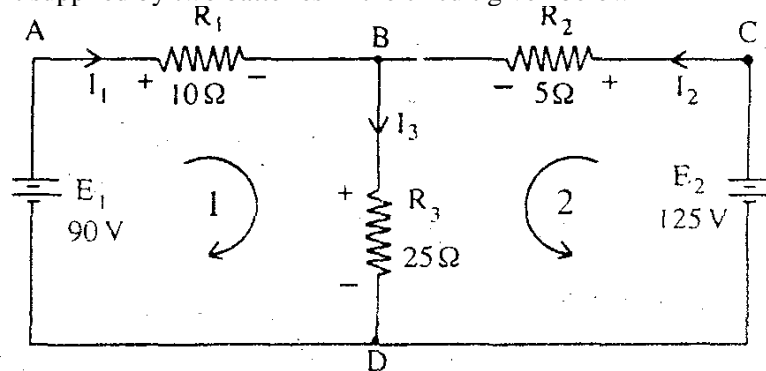
$$I_1 - I_2 + I_3 = 0$$

With the three equations, one can solve for the three currents I_1 , I_2 , and I_3 .

If the results obtained for I_1 , I_2 , and I_3 are all positive, then the assumed direction of the currents are said to be along the actual directions. A negative result for one or more currents will indicate that the assumed direction of the respective current is opposite to the actual direction.

Example 1.11

Calculate the current supplied by two batteries in the circuit given below



Solution:

The four junctions are marked as A, B, C and D. The current through R_1 is assumed to flow from A to B and through R_2 , from C to B and finally through R_3 from B to D. With reference to current directions, polarities of the voltage drop in R_1 , R_2 and R_3 are then marked as shown in the figure. Applying KCL to junction B

$$I_3 = I_1 + I_2 \quad \dots\dots(1)$$

Applying KVL to loop 1

$$\begin{aligned} E_1 - I_1 R_1 - I_3 R_3 &= 0 \\ I_1 R_1 + I_3 R_3 &= E_1 \\ 10I_1 + 25I_3 &= 90 \quad \dots\dots (2) \end{aligned}$$

Substituting Eq. (1) in Eq. (2)

$$\begin{aligned} 10I_1 + 25(I_1 + I_2) &= 90 \\ 35I_1 + 25I_2 &= 90 \quad \dots\dots (3) \end{aligned}$$

Applying KVL to loop 2

$$\begin{aligned} E_2 - I_2 R_2 - I_3 R_3 &= 0 \\ I_2 R_2 + I_3 R_3 &= E_2 \\ 5I_2 + 25I_3 &= 125 \quad \dots\dots (4) \end{aligned}$$

Substituting Eq. (1) in Eq. (4)

$$\begin{aligned} 5I_2 + 25(I_1 + I_2) &= 125 \\ 25I_1 + 30I_2 &= 125 \quad \dots\dots (5) \end{aligned}$$

Multiplying Eq. (3) by 6/5 we get

$$42I_1 + 30I_2 = 108 \quad \dots\dots (6)$$

Subtracting Eq. (6) from Eq. (5)

$$\begin{aligned} -17I_1 &= 17 \\ I_1 &= -1 \text{ A} \end{aligned}$$

Substituting the value of I_1 in Eq. (5) we get

$$I_2 = 5 \text{ A}$$

As the sign of the current I_1 is found to be negative from the solution, the actual direction of I_1 is from B to A to D i.e. 90 V battery gets a charging current of 1 A.

1.8 RESISTIVE NETWORKS

1.8.1 SERIES RESISTORS AND VOLTAGE DIVISION

Two or more resistors are said to be in series if the same current flows through all of them. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 1.18.

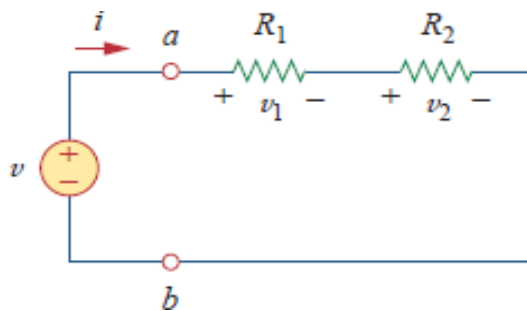


Fig.1.18 A single loop circuit with two resistors in series

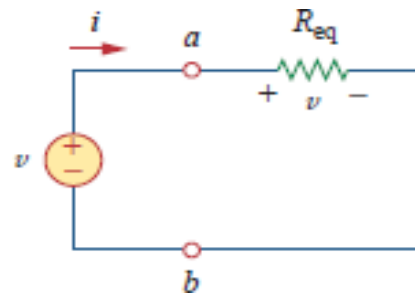


Fig. 1.19 Equivalent Circuit of series resistors

The two resistors are in series, since the same current i flow in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, v_2 = iR_2 \quad \dots\dots\dots (1)$$

If we apply KVL to the loop (moving in the clockwise direction), we have

$$v - v_1 - v_2 = 0 \quad \dots\dots\dots (2)$$

Combining equations (1) and (2), we get

$$v = v_1 + v_2 = i(R_1 + R_2) \quad \dots\dots\dots (3)$$

Or

$$i = \frac{v}{R_1 + R_2} \quad i = \frac{v}{R_1 + R_2} \quad \dots\dots\dots (4)$$

Equation (3) can be written $\dots\dots\dots (5)$

$$v = iR_{eq}$$

as

implying that the two resistors can be replaced by an equivalent resistor ;that is

$$R_{eq} = R_1 + R_2 \quad \dots\dots\dots (6)$$

Thus, Fig. 1.18 can be replaced by the equivalent circuit in Fig. 1.19. The two circuits in Fig 1.18 and 1.19 are the equivalent because they exhibit the same voltage-current relationships at the terminals a-b. An equivalent circuit such as the one in Fig. 1.19 is useful in simplifying the analysis of a circuit.

In general, the equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N = \sum_{n=1}^N R_n \quad \dots\dots\dots (7)$$

VOLTAGE DIVISION:

To determine the voltage across each resistor in Fig. 1.18, we substitute Eq. (4) into Eq. (1) and obtain

$$v_1 = \frac{v}{R_1 + R_2} R_1, v_2 = \frac{v}{R_1 + R_2} R_2 \quad \dots\dots\dots (8)$$

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the principle of voltage division, and the circuit in Fig. 1.18 is called a voltage divider. In general, if a voltage divider has N resistors (R_1, R_2, \dots, R_N) in series with the source voltage v , the nth resistor (R_N) will have a voltage drop of

$$v_N = \frac{R_N}{R_1 + R_2 + \dots + R_N} v \quad \dots\dots\dots (9)$$

1.8.2 PARALLEL RESISTORS AND CURRENT DIVISION

Two or more resistors are said to be in parallel if the same voltage appears across each element. Consider the circuit in Fig. 1.20, where two resistors are connected in parallel and therefore have the same voltage across them.

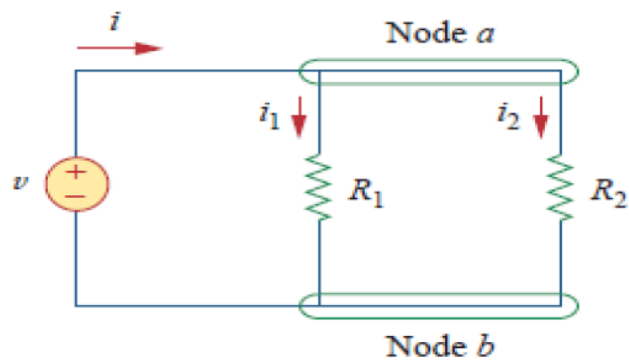


Fig. 1.20 Two resistors in parallel

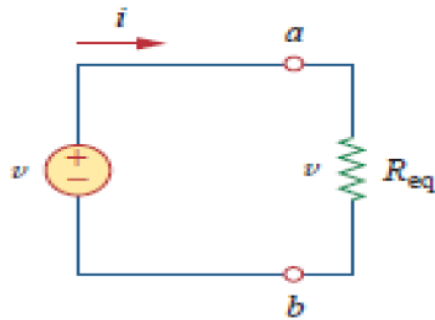


Fig. 1.21 Equivalent circuit of Fig. 1.20

$$v = i_1 R_1 = i_2 R_2 \quad \dots\dots\dots (1)$$

$$i_1 = \frac{v}{R_1}, i_2 = \frac{v}{R_2} \quad \dots\dots\dots (2)$$

Applying KCL at node *a* gives the total current *i* as

$$i = i_1 + i_2 \quad \dots\dots\dots (3)$$

Substituting Eq. (2) into Eq. (3), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v}{R_{eq}} \quad \dots\dots\dots (4)$$

where *R_{eq}* is the equivalent resistance of the resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \dots\dots\dots (5)$$

Or

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \dots\dots\dots (6)$$

Thus,

The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.

It must be emphasized that this applies only to two resistors in parallel. From Eq. (6), if *R₁* = *R₂*, then *R_{eq}* = *R₁*/2.

We can extend the result in Eq. (5) to the general case of a circuit with *N* resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} = \sum_{n=1}^N \frac{1}{R_n} \dots\dots\dots (7)$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}}$$

Thus,

The equivalent Resistance of parallel-connected resistors is the reciprocal of the sum of the reciprocals of the individual resistances.

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination.

Current Division:

Given the total current i entering node a in Fig. 1.20, then how do we obtain currents i_1 and i_2 ? We know that the equivalent resistor has the same voltage, or

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2} \dots\dots\dots (8)$$

Substitute (8) in (2), we get

$$i_1 = \frac{iR_2}{R_1 + R_2}$$

$$i_2 = \frac{iR_1}{R_1 + R_2} \dots\dots\dots (9)$$

This shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit in Fig.1.20 is known as a current divider. Notice that the larger current flows through the smaller resistance.

1.9 INDUCTIVE NETWORKS

Now that the inductor has been added to our list of passive elements, it is Necessary to extend the powerful tool of series-parallel combination. We need to know how to find the equivalent inductance of a series-connected or parallel-connected set of inductors found in practical circuits.

1.9.1 SERIES INDUCTORS

Two or more inductors are said to be in series, if the same current flows through all of them. Consider a series connection of N inductors, as shown in Fig. 1.22(a), with the equivalent circuit shown in Fig. 1.22(b). The inductors have the same current through them.

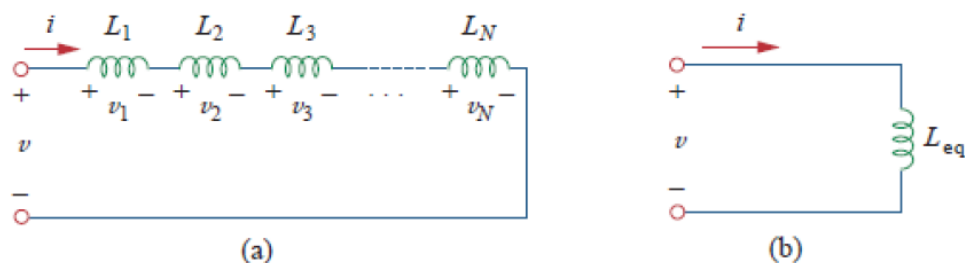


Fig. 1.22 (a) series connection of N inductors (b) Equivalent circuit for the series inductors

Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \dots + v_N \dots\dots\dots (1)$$

We know that the voltage across an inductor is $v = L \frac{di}{dt}$

Therefore, Eq. (1) becomes

$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \dots + L_N \frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \dots + L_N) \frac{di}{dt} \dots \dots \dots (2)$$

$$= \sum_{n=1}^N (L_n) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

Where,

$$L_{eq} = (L_1 + L_2 + L_3 + \dots + L_N \dots \dots \dots (3)$$

Thus

The equivalent inductance of series-connected inductors is the sum of the individual inductances.

* Inductors in series are combined in exactly the same way as resistors in series.

1.9.2 INDUCTORS IN PARALLEL

Two or more inductors are said to be in parallel, if the same voltage appears across each element. We now consider a parallel connection of N inductors, as shown in Fig. 1.23(a), with the equivalent circuit in Fig. 1.23(b). The inductors have the same voltage across them.

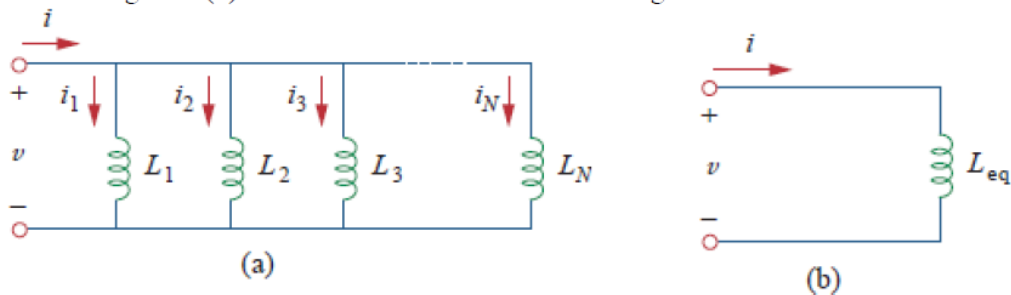


Fig. 1.23 (a) Parallel connection of N inductors (b) Equivalent circuit for parallel inductors

Using KCL,

$$i = i_1 + i_2 + i_3 + \dots + i_N \dots \dots \dots (1)$$

But the current through the inductor is

$$i = \frac{1}{L} \int_0^t v dt + i(0)$$

If we neglect the initial value of current i.e, $i(0) = 0$ then current through inductor becomes

$$i = \frac{1}{L} \int_0^t v dt$$

Hence,

$$i = \frac{1}{L_1} \int_0^t v dt + \frac{1}{L_2} \int_0^t v dt + \frac{1}{L_3} \int_0^t v dt + \dots + \frac{1}{L_N} \int_0^t v dt$$

$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N} \right) \int_0^t v dt$$

$$\therefore i = \left(\sum_{n=1}^N \frac{1}{L_n} \right) \int_0^t v dt = \frac{1}{L_{eq}} \int_0^t v dt \dots \dots \dots (2)$$

Where,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N}$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N}}$$

Thus,

The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

* Note that the inductors in parallel are combined in the same way as resistors in parallel.

1.10 CAPACITIVE NETWORKS

We know from resistive circuits and inductive circuits that the series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor C_{eq} .

1.10.1 SERIES CAPACITORS

Two or more capacitors are said to be in series, if the same current flows through all of them. Consider a series connection of N capacitors, as shown in Fig. 1.24(a), with the equivalent circuit shown in Fig. 1.24(b). The capacitors have the same current through them.

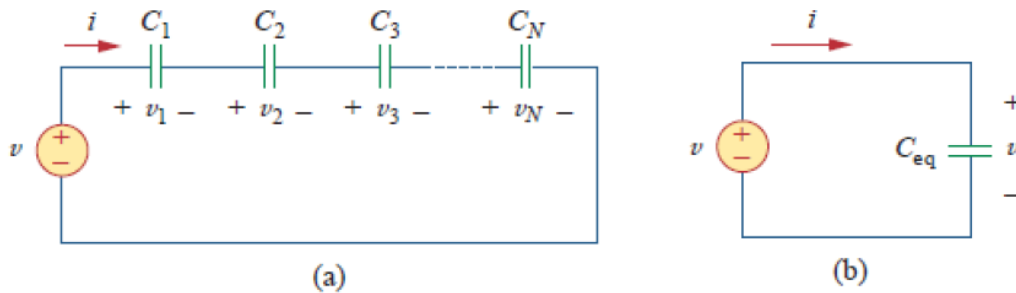


Fig. 1.24 (a) series connection of N capacitors (b) Equivalent circuit for the series capacitors

Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \dots + v_N \dots \dots (1)$$

We know that the voltage across a capacitor is

$$v = \frac{1}{C} \int_0^t i dt + v(0)$$

If we neglect the initial value of voltage i.e. $v(0) = 0$ then voltage across the capacitor becomes

$$v = \frac{1}{C} \int_0^t i dt$$

Hence, Eq. (1) becomes

$$v = \frac{1}{C_1} \int_0^t idt + \frac{1}{C_2} \int_0^t idt + \frac{1}{C_3} \int_0^t idt + \dots + \frac{1}{C_N} \int_0^t idt$$

$$v = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N} \right) \int_0^t idt$$

$$\therefore v = \left(\sum_{n=1}^N \frac{1}{C_n} \right) \int_0^t idt = \frac{1}{C_{eq}} \int_0^t idt \dots \dots (2)$$

Where,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}}$$

Thus,

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

* Note that the capacitors in series are combined in the same way as resistors in parallel.

For N=2,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

1.10.2 PARALLEL CAPACITORS

Two or more capacitors are said to be in parallel, if the same voltage appears across each element. Consider a parallel connection of N capacitors, as shown in Fig. 1.25(a), with the equivalent circuit in Fig. 1.25(b). The capacitors have the same voltage across them.

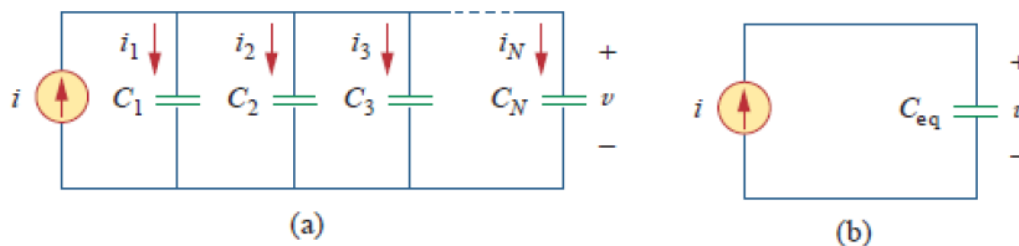


Fig. 1.25 (a) Parallel connection of N capacitors (b) Equivalent circuit for parallel capacitors

Applying KCL to Fig. 1.25(a)

$$i = i_1 + i_2 + i_3 + \dots + i_N \dots \dots (1)$$

We know that the current through capacitor is

$$i = C \frac{dv}{dt}$$

Therefore, Eq. (1) becomes

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + \dots + C_N \frac{dv}{dt}$$

$$= (C_1 + C_2 + C_3 + \dots + C_N) \frac{dv}{dt} \dots \dots \dots (2)$$

$$= \sum_{n=1}^N (C_n) \frac{dv}{dt} = C_{eq} \frac{dv}{dt}$$

Where,

$$C_{eq} = (C_1 + C_2 + C_3 + \dots + C_N) \dots \dots \dots (3)$$

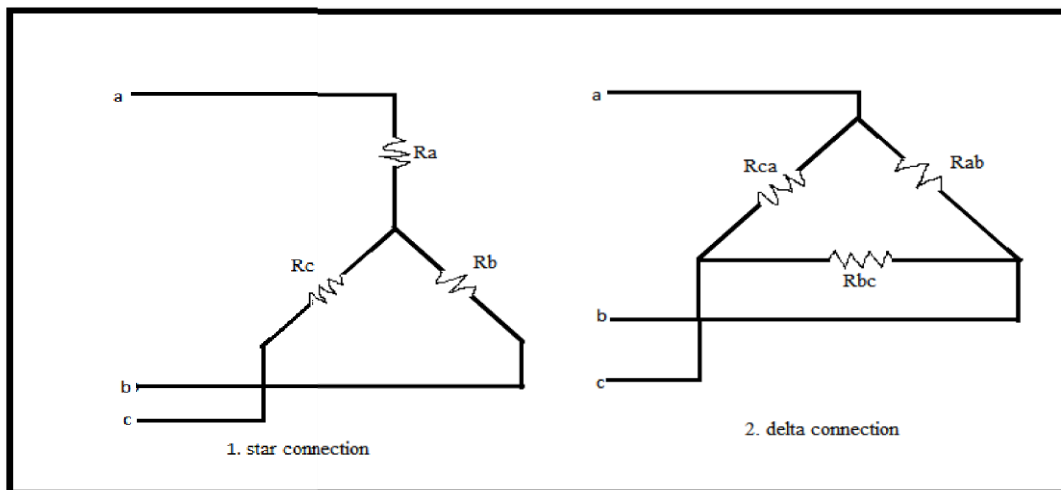
Thus

The equivalent capacitance of parallel-connected capacitors is the sum of the individual capacitances.

* Capacitors in parallel are combined in exactly the same way as resistors in series.

STAR – DELTA AND DELTA – STAR TRANSFORMATION

If there are three resistances are connected to a common point in the form as shown in fig (1). They are said to be star connected and if they are connected as shown in fig (2) they are said to be delta connected.



- In order to reduce the networks, it may be necessary to replace a star connected set of resistances by an equivalent delta connected set of resistances vice versa.
- The star delta transformation technique is useful in solving complex networks. Basically, any three circuit elements, i.e. Resistive, Inductive or capacitive, may be connected in two different ways. One way of connecting these elements is called the star connection, or the Y connection. The other way of connecting these elements is called delta connection or Δ connection.

The equivalence between the above two networks is obtained by equating the effective resistance between the corresponding terminals for the two networks.

Equating the resistances between corresponding pairs of terminals, Between a & b

$$R_a + R_b = R_{ab} (R_{bc} + R_{ca}) / (R_{ab} + R_{bc} + R_{ca}) \dots \dots \dots (1)$$

Between b & c

$$R_b + R_c = R_{bc} (R_{ca} + R_{ab}) / (R_{ab} + R_{bc} + R_{ca}) \text{-----} (2)$$

Between c & a

$$R_c + R_a = R_{ca} (R_{ab} + R_{bc}) / (R_{ab} + R_{bc} + R_{ca}) \text{-----}(3)$$

Subtracting (2) from (1) we get

$$R_a - R_c = R_{ca} (R_{ab} - R_{bc}) / (R_{ab} + R_{bc} + R_{ca}) \text{-----}(4)$$

Adding (3) and (4) we get

$$R_a = R_{ca} \cdot R_{ab} / (R_{ab} + R_{bc} + R_{ca}) \text{-----} (5)$$

Similarly,

$$R_b = R_{bc} \cdot R_{ab} / (R_{ab} + R_{bc} + R_{ca}) \text{-----}(6)$$

$$R_c = R_{ca} \cdot R_{bc} / (R_{ab} + R_{bc} + R_{ca}) \text{-----} (7)$$

Equations (5), (6) & (7) to transform delta – star i.e. we can obtain an equivalent star connected resistances for the given delta connected resistances.

From the above equations, we get

$$R_a R_b + R_b R_c + R_c R_a = R_{ab} \cdot R_{bc} \cdot R_{ca} / (R_{ab} + R_{bc} + R_{ca}) \text{-----} (8)$$

Dividing (8) by R_a i.e. equation (5)

$$R_a R_b + R_b R_c + R_c R_a = R_{bc}$$

. R_a Cancel R_a on both sides & we get

$$R_b + R_c + (R_b \cdot R_c / R_a) = R_{bc} \text{-----} (9)$$

Similarly dividing equation (8) by equation R_b & R_c , we got

$$R_a + R_b + (R_a \cdot R_b / R_c) = R_{ab} \text{-----} (10)$$

$$R_c + R_a + (R_a \cdot R_c / R_b) = R_{ca} \text{-----} (11)$$

From the above equations (9),(10) ,(11) we can replace a star connected resistances by an equivalent delta connected resistances.

5.2. Maxwell's Loop (or Mesh) Current Method

The method of *loop* or *mesh* currents is generally used in solving networks having some degree of complexity. Such a degree of complexity already begins for a network of three meshes. It might even be convenient at times to use the method of loop or mesh currents for solving a two-mesh circuit.

The *mesh-current method* is preferred to the general or branch-current method because the unknowns in the initial stage of solving a network are equal to the number of meshes, *i.e.*, the mesh currents. The necessity of writing the node-current equations, as done in the general or branch-current method where branch currents are used, is *obviated*. There are as many mesh-voltage equations as there are independent loop or mesh currents. Hence, the M-mesh currents are obtained by solving the M-mesh voltages or loop equations for M unknowns. After solving for the mesh currents, only a matter of resolving the confluent mesh currents into the respective branch currents by very simple algebraic manipulations is required.

This method eliminates a great deal of tedious work involved in branch-current method and is best suited when energy sources are voltage sources rather than current sources. This method can be used only for planar circuits.

The **procedure** for writing the equations is as follows :

1. Assume the smallest number of mesh currents so that at least one mesh current links every element. As a matter of convenience, all mesh currents are assumed to have a *clockwise direction*.

The number of mesh currents is equal to the number of meshes in the circuit.

2. For each mesh write down the Kirchhoff's voltage law equation. Where more than one mesh current flows through an element, the algebraic sum of currents should be used. The algebraic sum of mesh currents may be sum or the difference of the currents flowing through the element depending on the direction of mesh currents.

3. Solve the above equations and from the mesh currents find the branch currents.

Fig. 46 shows two batteries E_1 and E_2 connected in a network consisting of three resistors. Let the loop currents for two meshes be I_1 and I_2 (both clockwise-assumed). It is obvious that current through R_3 (when considered as a part of first loop) is $(I_1 - I_2)$. However, when R_3 is considered part of the second loop, current through it is $(I_2 - I_1)$.

Applying Kirchhoff's voltage law to the *two loops*, we get

$$\begin{array}{l}
 \text{or} \\
 \text{Similarly,} \\
 \text{or}
 \end{array}
 \begin{array}{l}
 E_1 - I_1 R_1 - R_3(I_1 - I_2) = 0 \\
 E_1 - I_1(R_1 + R_3) + I_2 R_3 = 0 \quad \dots \text{Loop 1} \\
 -I_2 R_2 - E_2 - R_3(I_2 - I_1) = 0 \\
 -I_2 R_2 - E_2 - I_2 R_3 + I_1 R_3 = 0 \\
 I_1 R_3 - I_2(R_2 + R_3) - E_2 = 0 \quad \dots \text{Loop 2}
 \end{array}$$

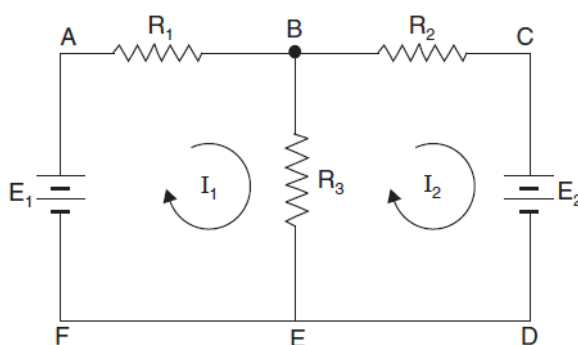


Fig. 46

The above two equations can be solved not only to find loop currents but branch currents as well.

Example 33. Determine the currents through various resistors of the circuit shown in Fig. 47 using the concept of mesh currents.

Solution. Refer Fig. 47.

Since there are two meshes, let the loop currents be as shown.

Applying Kirchhoff's law to loop 1, we get

$$24 - 4I_1 - 2(I_1 - I_2) = 0$$

$$-6I_1 + 2I_2 + 24 = 0$$

or $3I_1 - I_2 = 12 \quad \dots(i)$

For loop 2, we have

$$-2(I_2 - I_1) - 6I_2 - 12 = 0$$

$$2I_1 - 8I_2 - 12 = 0$$

$$I_1 - 4I_2 = 6 \quad \dots(ii)$$

Solving (i) and (ii), we get $I_1 = \frac{42}{11}$ A

and $I_2 = -\frac{6}{11}$ A

Hence Current through 4 Ω resistor = $\frac{42}{11}$ A (from L to M). (Ans.)

Current through 6 Ω resistor = $\frac{6}{11}$ A (from N to M). (Ans.)

Current through 2 Ω resistor = $\frac{42}{11} - \left(-\frac{6}{11}\right) = \frac{48}{11}$ A (from M to P). (Ans.)

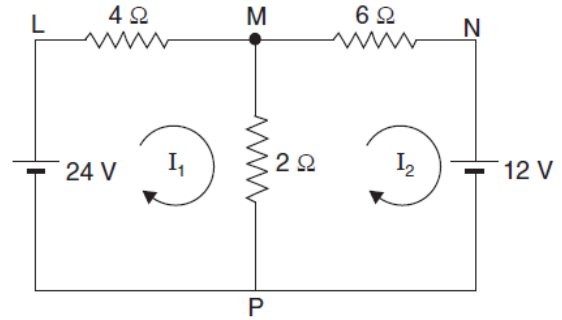


Fig. 47

Example 34. Determine the current supplied by each battery in the circuit shown in Fig. 48.

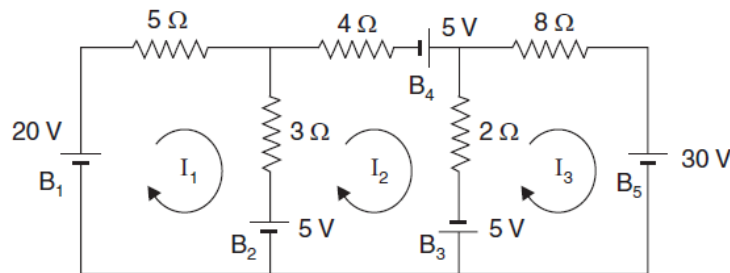


Fig. 48

Solution. Refer to Fig. 48.

As there are three meshes, let the three loop currents be as shown.

Applying Kirchhoff's law to loop 1, we get

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$$

or $8I_1 - 3I_2 = 15 \quad \dots(i)$

For loop 2, we have

$$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$$

$$3I_1 - 9I_2 + 2I_3 = -15 \quad \dots(ii)$$

For loop 3, we have

$$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$$

$$2I_2 - 10I_3 = 35 \quad \dots(iii)$$

Eliminating I_1 from (i) and (ii), we get

$$63I_2 - 16I_3 = 165 \quad \dots(iv)$$

Solving (iii) and (iv), we get

$$I_2 = 1.82 \text{ A} \quad \text{and} \quad I_3 = -3.15 \text{ A}$$

(- ve sign means direction of current is counter-clockwise)

Substituting the value of I_2 in (i), we get

$$I_1 = 2.56 \text{ A}$$

Current through battery B_1 (discharging current) = $I_1 = 2.56 \text{ A}$. (Ans.)

Current through battery B_2 (charging current) = $I_1 - I_2 = 2.56 - 1.82 = 0.74 \text{ A}$. (Ans.)

Current through battery B_3 (discharging current) = $I_2 + I_3 = 1.82 + 3.15 = 4.97 \text{ A}$. (Ans.)

Current through battery B_4 (discharging current) = $I_2 = 1.82 \text{ A}$. (Ans.)

Current through battery B_5 (discharging current) = $I_3 = 3.15 \text{ A}$. (Ans.)

Example 35. Determine the currents through the different branches of the bridge circuit shown in Fig. 49.

Solution. Refer to Fig. 49.

The three mesh currents are assumed as shown.

The equations for the three meshes are :

$$\text{For loop 1 : } 240 - 20(I_1 - I_2) - 50(I_1 - I_3) = 0$$

$$\text{or } -70I_1 + 20I_2 + 50I_3 = -240$$

$$\text{or } 70I_1 - 20I_2 - 50I_3 = 240 \quad \dots(i)$$

$$\text{For loop 2 : } -30I_2 - 40(I_2 - I_3) - 20(I_2 - I_1) = 0$$

$$\text{or } 20I_1 - 90I_2 + 40I_3 = 0$$

$$\text{or } 2I_1 - 9I_2 + 4I_3 = 0 \quad \dots(ii)$$

$$\text{For loop 3 : } -60I_3 - 50(I_3 - I_1) - 40(I_3 - I_2) = 0$$

$$50I_1 + 40I_2 - 150I_3 = 0$$

$$5I_1 + 4I_2 - 15I_3 = 0 \quad \dots(iii)$$

Solving these equations, we get

$$I_1 = 6.10 \text{ A}, I_2 = 2.56 \text{ A}, I_3 = 2.72 \text{ A}$$

Current through 30Ω resistor = I_2

$$= 2.56 \text{ A (A to B)}. \text{ (Ans.)}$$

Current through 60Ω resistor = $I_3 = 2.72 \text{ A (B to C)}. \text{ (Ans.)}$

Current through 20Ω resistor = $I_1 - I_2 = 6.10 - 2.56 = 3.54 \text{ A (A to D)}. \text{ (Ans.)}$

Current through 50Ω resistor = $I_1 - I_3 = 6.10 - 2.72 = 3.38 \text{ A (D to C)}. \text{ (Ans.)}$

Current through 40Ω resistor = $I_3 - I_2 = 2.72 - 2.56 = 0.16 \text{ A (D to B)}. \text{ (Ans.)}$

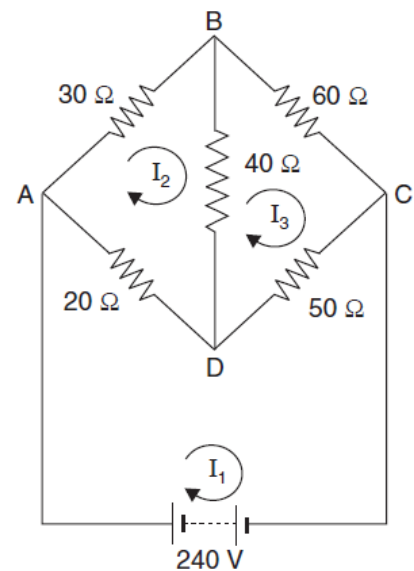


Fig. 49

5.3. Nodal Voltage Method

Under this method the following *procedure* is adopted :

1. Assume the voltages of the different independent nodes.
2. Write the equations for each node as per Kirchhoff's current law.
3. Solve the above equations to get the node voltages.
4. Calculate the branch currents from the values of node voltages.

Let us consider the circuit shown in the Fig. 52. L and M are the two independent nodes ; M can be taken as the reference node. Let the voltage of node L (with respect to M) be V_L .

Using Kirchoff's law, we get

$$I_1 + I_2 = I_3 \quad \dots(22)$$

Ohm's law gives
$$I_1 = \frac{V_1}{R_1} = \frac{(E_1 - V_L)}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{(E_2 - V_L)}{R_2} \quad \dots(23)$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_L}{R_3}$$

$$\frac{E_1 - V_L}{R_1} + \frac{E_2 - V_L}{R_2} = \frac{V_L}{R_3} \quad \dots(24)$$

Rearranging the terms, we get

$$V_L \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0 \quad \dots(25)$$

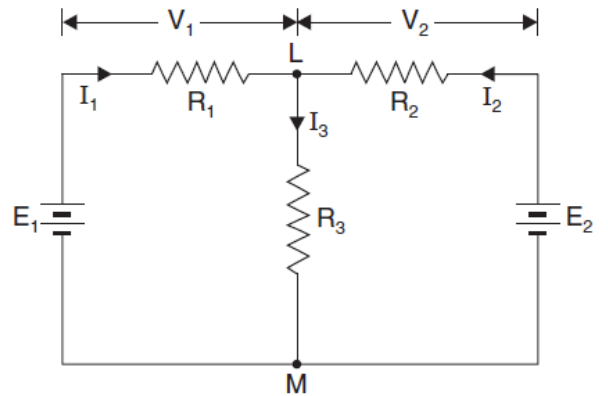


Fig. 52

It may be noted that the above nodal equation contains the following terms :

(i) The node voltage multiplied by the sum of all conductances connected to that anode. This term is *positive*.

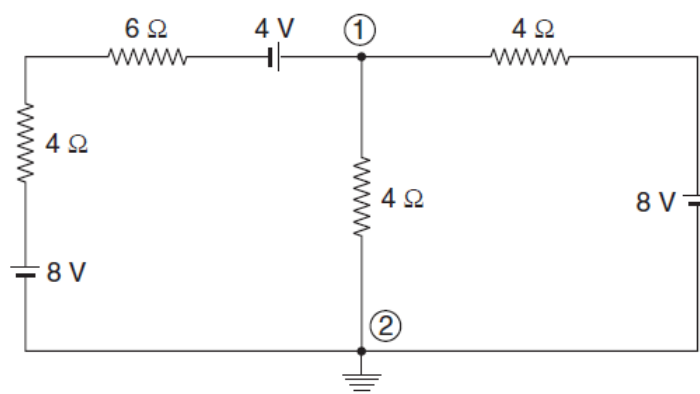
(ii) The node voltage at the other end of each branch (connected to this node) multiplied by the conductance of branch. These terms are *negative*.

— In this method of solving a network the *number of equations required for the solution is one less than the number of independent nodes in the network*.

— In general the nodal analysis *yields similar solutions*.

— The nodal method is very suitable for *computer work*.

Example 38. Using Node voltage method, find the current in 6Ω resistance for the network shown in Fig. 54.



Solution. Refer Fig. 54. Considering node 2 as the reference node and using node voltage method, we have

$$V_1 \left[\frac{1}{(6+4)} + \frac{1}{4} + \frac{1}{4} \right] - \frac{8}{4} - \left(\frac{8+4}{10} \right) = 0$$

(The reason for adding the two battery voltages of 4 V and 8 V is because they are connected in additive series).

or
$$V_1 (0.1 + 0.25 + 0.25) - 2 - 1.2 = 0$$

$$\therefore V_1 = 5.33 \text{ V}$$

The current flowing through the 6Ω resistance towards node 1 is

$$= \frac{12 - 5.33}{6 + 4} = \mathbf{0.667 \text{ A. (Ans.)}}$$

Example 37. For the circuit shown in Fig. 53, find the currents through the resistances R_3 and R_4 .

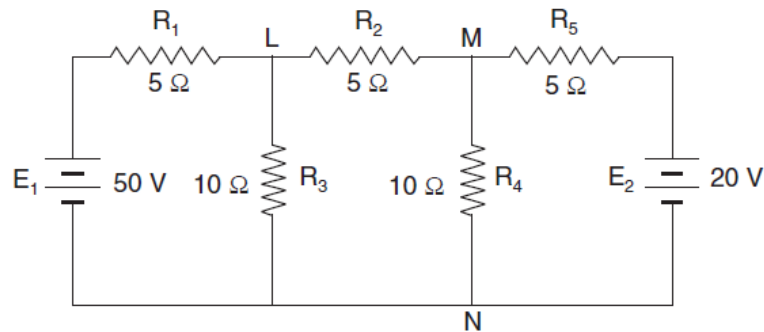


Fig. 53

Solution. Refer Fig. 53.

Let L , M and N = Independent nodes, and

V_L and V_M = Voltages of nodes L and M with respect to node N .

The nodal equations for the nodes L and M are :

$$V_L \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right] - \frac{E_1}{R_1} - \frac{V_M}{R_2} = 0 \quad \dots(i)$$

$$V_M \left[\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} \right] - \frac{E_2}{R_5} - \frac{V_L}{R_2} = 0 \quad \dots(ii)$$

Substituting the values in (i) and (ii) and simplifying, we get

$$V_L \left(\frac{1}{5} + \frac{1}{5} + \frac{1}{10} \right) - \frac{50}{5} - \frac{V_M}{5} = 0$$

or $2.5V_L - V_M - 50 = 0 \quad \dots(iii)$

and

$$V_M \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{5} \right) - \frac{20}{5} - \frac{V_L}{5} = 0$$

or $2.5V_M - V_L - 20 = 0$

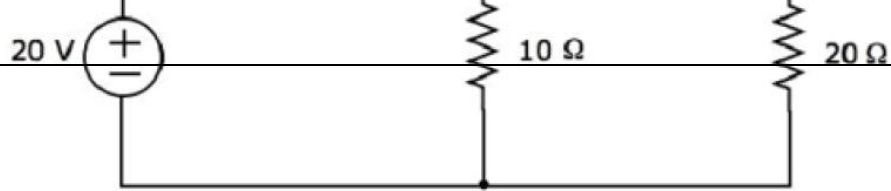
or $-V_L + 2.5V_M - 20 = 0 \quad \dots(iv)$

Solving (iii) and (iv), we get

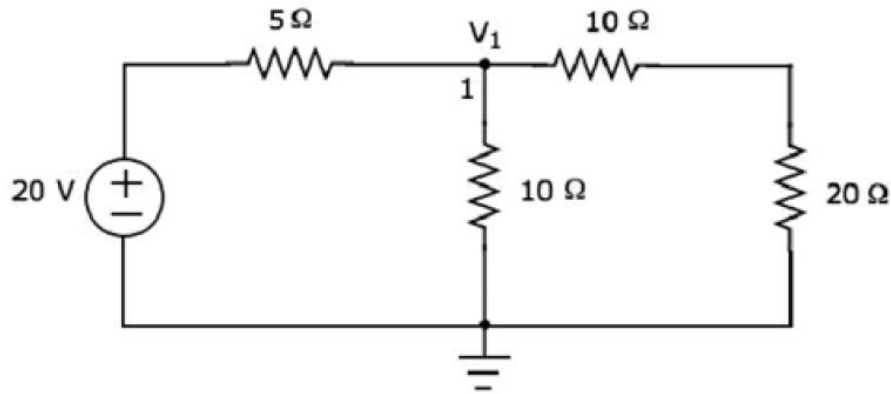
$$V_L = 27.6 \text{ V}, V_M = 19.05 \text{ V}$$

$$\text{Current through } R_3 = \frac{V_L}{R_3} = \frac{27.6}{10} = \mathbf{2.76 \text{ A. (Ans.)}}$$

$$\text{Current through } R_4 = \frac{V_M}{R_4} = \frac{19.05}{10} = \mathbf{1.905 \text{ A. (Ans.)}}$$



There is only one principal node except Ground in the above circuit. So, we can use **nodal analysis** method. The node voltage V_1 is labelled in the following figure. Here, V_1 is the voltage from node 1 with respect to ground.



The **nodal equation** at node 1 is

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1}{10 + 20} = 0$$

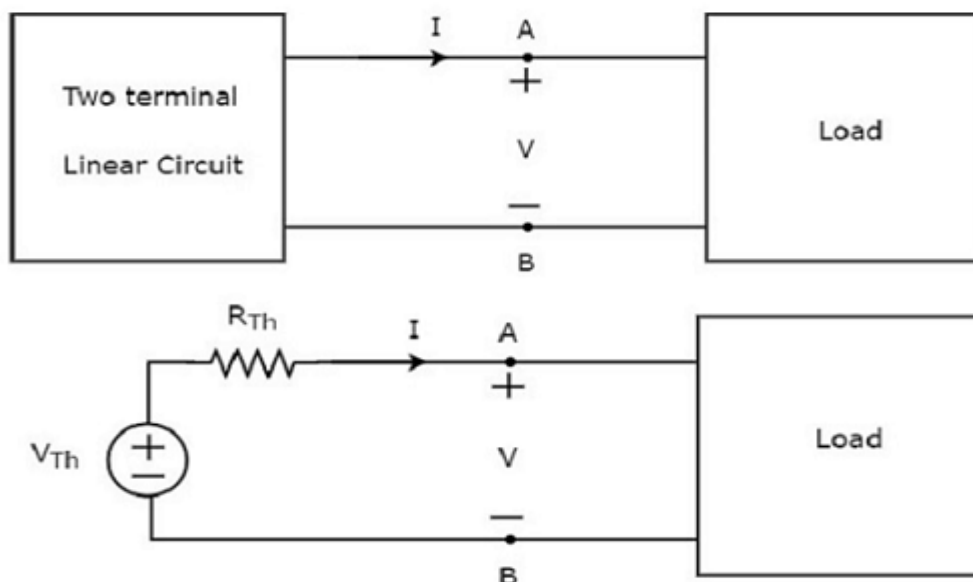
Thevenin's Theorem :

STATEMENT : It states that any two terminal linear network or circuit can be represented with an equivalent network or circuit, which consists of a voltage source in series with a resistor. It is known as Thevenin's equivalent circuit. A linear circuit may contain independent sources, dependent sources, and resistors.

If the circuit contains multiple independent sources, dependent sources, and resistors, then the response in an element can be easily found by replacing the entire network to the left of that element with a **Thevenin's equivalent circuit**.

The **response in an element** can be the voltage across that element, current flowing through that element, or power dissipated across that element.

This concept is illustrated in following figures.



Thevenin's equivalent circuit resembles a practical voltage source. Hence, it has a voltage source in series with a resistor.

- The voltage source present in the Thevenin's equivalent circuit is called as Thevenin's equivalent voltage or simply **Thevenin's voltage, V_{Th}** .
- The resistor present in the Thevenin's equivalent circuit is called as Thevenin's equivalent resistor or simply **Thevenin's resistor, R_{Th}** .

Methods of Finding Thevenin's Equivalent Circuit

There are three methods for finding a Thevenin's equivalent circuit. Based on the **type of sources** that are present in the network, we can choose one of these three methods. Now, let us discuss two methods one by one. We will discuss the third method in the next chapter.

Method 1

Follow these steps in order to find the Thevenin's equivalent circuit, when only the **sources of independent type** are present.

- **Step 1** – Consider the circuit diagram by opening the terminals with respect to which the Thevenin's equivalent circuit is to be found.
- **Step 2** – Find Thevenin's voltage V_{Th} across the open terminals of the above circuit.
- **Step 3** – Find Thevenin's resistance R_{Th} across the open terminals of the above circuit by eliminating the independent sources present in it.
- **Step 4** – Draw the **Thevenin's equivalent circuit** by connecting a Thevenin's voltage V_{Th} in series with a Thevenin's resistance R_{Th} .

Maximum power transfer theorem :

It states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.



UNIT II

AC CIRCUITS

1. INTRODUCTION TO ALTERNATING CURRENT

A.C. means **alternating current**—*The current or voltage which alternates its direction and magnitude every time.* Now a days 95% of the total energy is produced, transmitted and distributed in A.C. supply.

The reasons are the following :

- (i) More voltage can be generated (upto 33000 V) than D.C. (650 V only).
- (ii) A.C. voltage can be increased and decreased with the help of a static machine called the 'transformer'.
- (iii) A.C. transmission and distribution is more economical as line material (say copper) can be saved by transmitting power at higher voltage.
- (iv) A.C. motors for the same horse power as of D.C. motors are cheaper, lighter in weight, require less space and require lesser attention in operation and maintenance.
- (v) A.C. can be converted to D.C. (direct current) easily, when and where required but D.C. cannot be converted to A.C. so easily and it will not be economical.

However, D.C. entails the following *merits* and hence finds wide applications.

- (i) D.C. series motors are most suitable for traction purposes in tramway, railways, crains and lifts.
- (ii) For electroplating, electrolytic and electrochemical processes (battery charging etc.), D.C. is required.
- (iii) Arc lamps for search lights and cinema projectors work on D.C.
- (iv) Arc welding is better than on A.C.
- (v) Relay and operating time switches, etc., and circuit-breakers, D.C. works more efficiently.
- (vi) In rolling mills, paper mills, colliery winding, etc., where fine speed control of speeds in both directions is required, D.C. motors are required.

2. GENERATION AND EQUATIONS OF ALTERNATING VOLTAGES AND CURRENTS

Generation of Alternating Voltages and Currents

Alternating voltages may be generated in the following two ways :

1. By rotating a coil in a stationary magnetic field, as shown in Fig. 1.
2. By rotating a magnetic field within a stationary coil, as shown in Fig. 2.

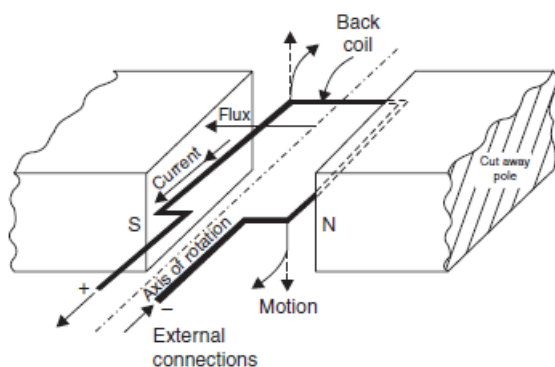


Fig. 1. Rotating a coil in a stationary magnetic field.

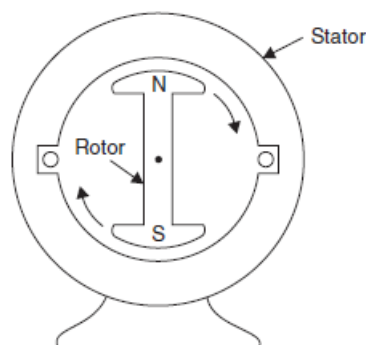


Fig. 2. Rotating a magnetic field within a stationary coil.

UNIT II AC CIRCUITS

The value of the voltage generated in each case depends upon the following factors :

- (i) The number of turns in the coils ;
 - (ii) The strength of the field ;
 - (iii) The speed at which the coil or magnetic field rotates.
- Out of the above two methods the *rotating-field method is mostly used in practice.*

Equations of Alternating Voltages and Currents

Fig. 3 shows a rectangular coil of N turns rotating clockwise with an angular velocity ω radians per second in a uniform magnetic field.

Since by Faraday's law, the voltage is proportional to the rate at which the conductor cuts across the magnetic field or to the rate of change of flux linkages, the shape of the wave of voltage applied to the external circuit will be determined by the *flux distribution in the air gap*. For a uniform field between the poles it is evident that *maximum flux* will link with the coil when its plane is in *vertical position i.e., perpendicular to the direction of flux* between the poles. Also it is obvious that when the plane of coil is *horizontal no flux will link with the coil*.

If the position of the coil with reference to the vertical axis be denoted by θ the flux linking with the coil at any instant, as the coil rotates may be determined from the relation,

$$\begin{aligned}\phi &= \phi_{max} \cos \theta \\ &= \phi_{max} \cos \omega t\end{aligned}\quad \dots(i) \quad (\because \theta = \omega t)$$

where, ϕ_{max} = Maximum flux which can link with the coil, and

t = Time taken by the coil to move through an angle θ from vertical position.

Using Faraday's law to eqn. (i), in order to determine the voltage equation,

$$e = -N \frac{d\phi}{dt} \quad (\text{where } e \text{ is the instantaneous value of the induced e.m.f.)}$$

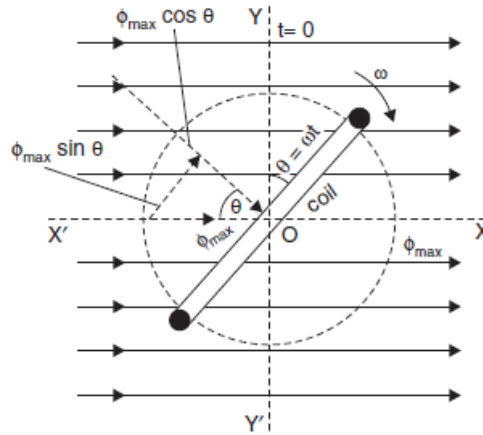


Fig. 3. A coil rotating in a magnet field.

UNIT II AC CIRCUITS

$$= -N \frac{d}{dt} (\phi_{max} \cos \omega t) = \omega N \phi_{max} \sin \omega t$$

or
$$e = \omega N \phi_{max} \sin \theta \quad \dots(ii)$$

As the value of e will be maximum when $\sin \theta = 1$,

$$\therefore E_{max} = \omega N \phi_{max}$$

The eqn. (ii) can be written in simpler form as

$$e = E_{max} \sin \theta \quad \dots(iii)$$

Similarly the equation of induced alternating current (instantaneous value) is

$$i = I_{max} \sin \theta \quad (\text{if the load is resistive}) \quad \dots(iv)$$

Waveforms. A waveform (or wave-shape) is the shape of a curve obtained by plotting the instantaneous values of voltage or current as ordinate against time as abscissa.

Fig. 4 (a, b, c, d, e) shows irregular waveforms, but each cycle of current/voltage is an exactly replica of the previous one. Alternating e.m.fs and currents produced by machines usually both have positive and negative half waves, the same shape as shown. Fig. 4(f) represents a sine wave of A.C. This is the simplest possible waveform, and alternators are designed to give as nearly as possible a sine wave of e.m.f.

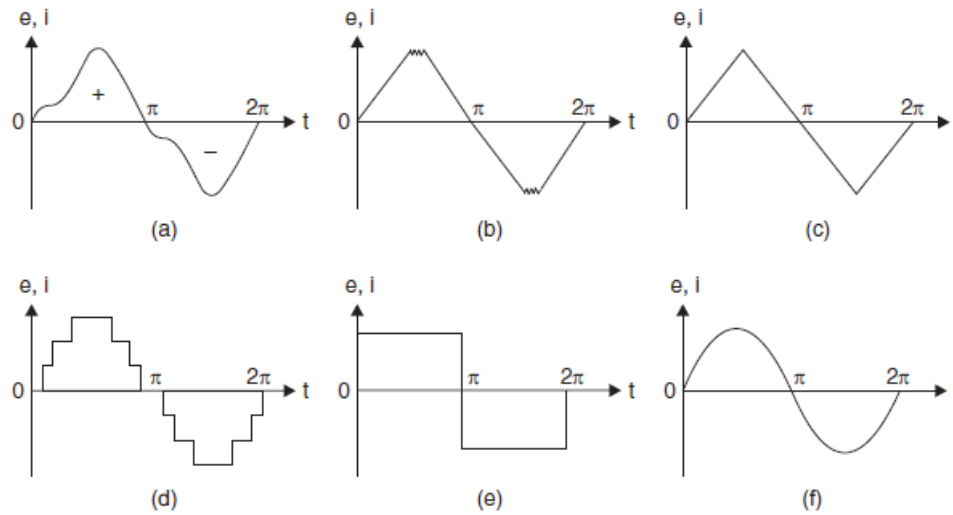


Fig. 4. Waveforms.

- In general, an *alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.*
- The waves deviating from the standard sine wave are termed as *distorted waves.*
- *Complex waves* are those which depart from the ideal sinusoidal form. All alternating complex waves, which are periodic and have equal positive and negative half cycles can be shown to be made up of a number of pure sine waves, having different frequencies but all these frequencies are integral multiples of that of the lowest alternating wave, called the *fundamental* (or first harmonic). These waves of higher frequencies are called *harmonics.*

UNIT II AC CIRCUITS

3. ALTERNATING VOLTAGE AND CURRENT

Modern alternators produce an e.m.f. which is for all practical purposes sinusoidal (*i.e.*, a sine curve), the equation between the e.m.f. and time being

$$e = E_{max} \sin \omega t \quad \dots(1)$$

where, e = Instantaneous voltage ; E_{max} = Maximum voltage ;

ωt = Angle through which the armature has turned from neutral.

Taking the frequency as f hertz (cycles per second), the value of ω will be $2\pi f$, so that the equation reads

$$e = E_{max} \sin (2\pi f)t.$$

The graph of the voltage will be as shown in Fig. 5.

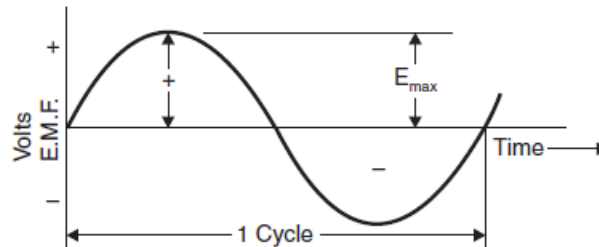


Fig. 5. The graph of the sinusoidal voltage.

1. Cycle. One complete set of positive and negative values of an alternating quantity is known as a *cycle*. A cycle may also sometimes be specified in terms of angular measure. In that case, one complete cycle is said to spread over 360° or 2π radians.

2. Amplitude. The maximum value, positive or negative, of an alternating quantity, is known as its *amplitude*.

3. Frequency (f). The number of cycles/second is called the frequency of the alternating quantity.

Its unit is *hertz* (Hz).

4. Time Period (T). The time taken by an alternating quantity to complete the cycle is called its *time period*. For example, a 50 hertz (Hz) alternating current has a time period of $\frac{1}{50}$ second.

Time period is reciprocal of frequency,

i.e.,
$$T = \frac{1}{f} \left(\text{or } f = \frac{1}{T} \right) \quad \dots(2)$$

5. Root mean square (R.M.S.) value. The r.m.s. (or effective) value of an alternating current is given by that steady (D.C.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

R.M.S. value is the value which is taken for power purposes of any description. This value is obtained by finding the square root of the mean value of the squared ordinates for a cycle or half-cycle (See Fig. 5).

This is the value which is used for all power, lighting and heating purposes, as in these cases the power is proportional to the square of the voltage.

Refer to Fig. 5.

UNIT II AC CIRCUITS

The equation of sinusoidal alternating current is given as :

$$i = I_{max} \sin \theta$$

The mean of squares of the instantaneous values of current over half cycle is

$$\begin{aligned} I^2 &= \int_0^\pi \frac{i^2 d\theta}{(\pi - 0)} \\ I^2 &= \frac{1}{\pi} \int_0^\pi i^2 d\theta = \frac{1}{\pi} \int_0^\pi (I_{max} \sin \theta)^2 d\theta \\ &= \frac{1}{\pi} \int_0^\pi I_{max}^2 \sin^2 \theta d\theta = \frac{I_{max}^2}{\pi} \int_0^\pi \left(\frac{1 - \cos 2\theta}{2} \right) d\theta \\ &= \frac{I_{max}^2}{2\pi} \int_0^\pi (1 - \cos 2\theta) d\theta = \frac{I_{max}^2}{2\pi} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^\pi \\ &= \frac{I_{max}^2}{2\pi} \times \pi = \frac{I_{max}^2}{2} \quad \text{or} \quad I = \sqrt{\frac{I_{max}^2}{2}} = \frac{I_{max}}{\sqrt{2}} \end{aligned}$$

or $I = 0.707 I_{max}$... (3)

Note. While solving problems, the values of given current and voltage should always be taken as the r.m.s. values, unless indicated otherwise.

6. Average or mean value. The average value of an alternating current is expressed by *that steady current which transfers across any circuit the same charge as is transferred by that alternating current during the same time.*

The mean value is only of use in connection with processes where the results depend on the current only, irrespective of the voltage, such as electroplating or battery charging.

Refer to Fig. 6.

The value of instantaneous current is given by

$$i = I_{max} \sin \theta$$

Refer to Fig. 6. The value of instantaneous current is given by :

$$i = I_{max} \sin \theta \quad [\theta = \omega t]$$

$$I_{av} = \frac{1}{(\pi - 0)} \int_0^\pi i d\theta$$

[Limits are taken from 0 to π , since only first half cycle is considered. For whole cycle, the average value of sine wave is zero.]

$$\begin{aligned} &= \frac{1}{\pi} \int_0^\pi I_{max} \cdot \sin \theta d\theta = \frac{1}{\pi} \cdot I_{max} \left[-\cos \theta \right]_0^\pi \\ &= \frac{1}{\pi} \cdot I_{max} [1 - (-1)] = \frac{2}{\pi} \cdot I_{max} \end{aligned}$$

or $I_{av} = 0.637 I_{max}$... (4)

Note. In case of unsymmetrical alternating current *viz.* half-wave rectified current the average value must always be taken over the whole cycle.

7. Form and Peak Factors

Form factor. The ratio of r.m.s. (or effective) value to average value is the form factor (K_f) of the wave form. It has use in voltage generation and instrument correction factors.

Peak factor. The ratio of maximum value to the r.m.s. value is the peak factor (K_p) of the wave form.

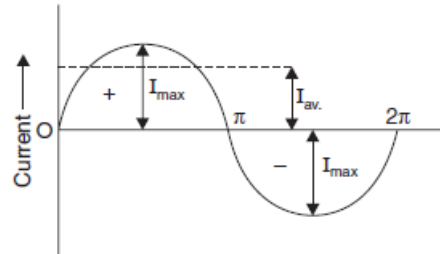


Fig. 6

UNIT II AC CIRCUITS

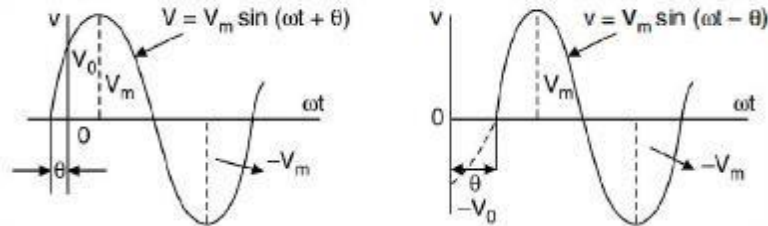
Phase and Phase difference:

Phase of the sine indicates starting phase of the sine wave. i.e

Let , $V(t) = V_m \sin \omega t$, here we can say that phase is zero as function starts from origin.

$V(t) = V_m \sin(\omega t - \theta)$, here we can say that phase of function is θ degrees to right shift.

$V(t) = V_m \sin(\omega t + \theta)$, here we can say that phase of function is θ degrees to left shift.



Phase difference is the difference of phase between two wave forms taking one as reference.

Eg: If wave form A is $V_m \sin(\omega t + 15)$, B is $V_m \sin(\omega t - 30)$ and C is $V_m \sin(\omega t + 45)$.

Determine the phase difference between every pair if wave forms. When A and B are compared , phase difference is 45 degrees.

When C and B are compared , phase difference is 75 degrees. When A and C are compared , phase difference is 30 degrees.

Phasor Diagram:

Phasor diagram is the pictorial representation of sine wave. Here magnitude and phase of the wave function are represented in four quadrant axis. We assume positive phases in anti-clock wise direction and negative phases in clock wise direction. From the phasor diagram we can easily identify the phase difference between different wave forms. We can also identify whether function is right shift or left shift.

UNIT II AC CIRCUITS

4. SINGLE PHASE CIRCUITS

The study of circuits involves three basic types of units (R, L, C i.e., resistance, reactance and capacitance respectively) and four possible series combination of them. The latter, in turn, may be arranged in many kinds of parallel, series-parallel, parallel-series or other complex circuits.

4.1. A.C. Through Pure Ohmic Resistance Alone

The circuit containing a pure resistance R is shown in Fig. 23 (a). Let the applied voltage be given by the equation,

$$v = V_{max} \sin \theta = V_{max} \sin \omega t \quad \dots(i)$$

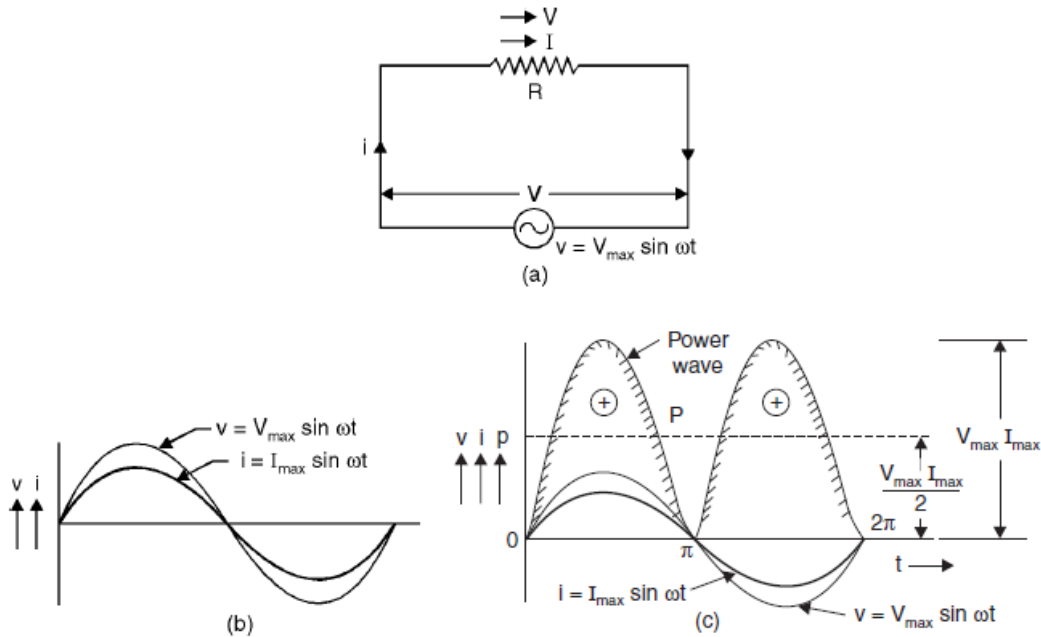


Fig. 23. A.C. through pure ohmic resistance alone.

Then the instantaneous value of current flowing through the resistance R will be,

$$i = \frac{v}{R} = \frac{V_{max} \sin \omega t}{R} \quad \dots(ii)$$

The value of current will be maximum

when $\sin \omega t = 1$ or $(\omega t = 90^\circ)$

$$\therefore I_{max} = \frac{V_{max}}{R}$$

Substituting this value in eqn. (ii), we get

$$i = I_{max} \sin \omega t \quad \dots(iii)$$

Comparing (i) and (iii), we find that alternating voltage and current are in phase with each other as shown in Fig. 23 (b), also shown vectorially in Fig. 23 (e).

UNIT II AC CIRCUITS

Power. Refer to Fig. 23 (c)

Instantaneous power,

$$\begin{aligned} p &= vi = V_{max} \sin \omega t \times I_{max} \sin \omega t = V_{max} I_{max} \sin^2 \omega t \\ &= \frac{V_{max} I_{max}}{2} \times 2 \sin^2 \omega t = \frac{V_{max} I_{max}}{2} (1 - \cos 2 \omega t) \\ &= \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} - \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos 2 \omega t \\ &\quad \text{(Constant part) (Fluctuating part)} \end{aligned}$$

For a complete cycle the average of $\frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos 2 \omega t$ is zero.

Hence, power for the whole cycle,

$$P = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} = V_{r.m.s.} \cdot I_{r.m.s.}$$

or

$$P = VI \text{ watt}$$

where V = R.M.S. value of applied voltage, and

I = R.M.S. value of the current.

It may be observed from the Fig. 23 (c) that no part of the power cycle at any time becomes negative. In other words the power in a purely resistive circuit *never becomes zero*.

Hence in *pure resistive circuit* we have :

1. Current is in phase with the voltage.
2. Current $I = \frac{V}{R}$ where I and V are r.m.s. values of current and voltage.
3. Power in the circuit, $P = VI = I^2R$.

4.2. A.C. Through Pure Inductance Alone

Fig. 24 (a) shows the circuit containing a pure inductance of L henry.

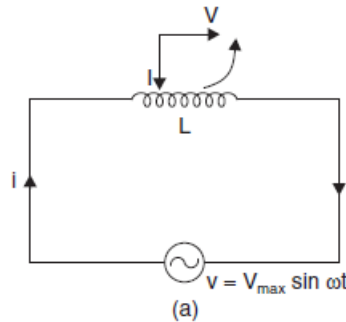
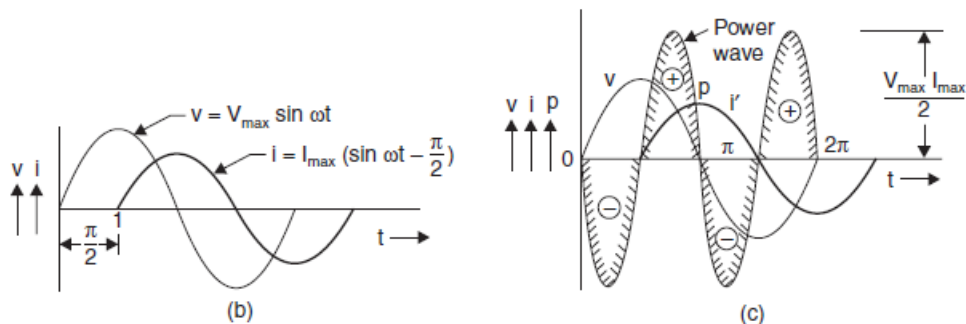


Fig. 24 (a)

Let the alternating voltage applied across the circuit be given by the equation,

$$v = V_{max} \sin \omega t \quad \dots(i)$$

Whenever an alternating voltage is applied to a purely inductive coil, a back e.m.f. is produced due to the self-inductance of the coil. This back e.m.f. opposes the rise or fall of the current through the coil. Since there is no ohmic drop in this case, therefore, the applied voltage has to overcome this induced e.m.f. only. Thus at every step,



UNIT II AC CIRCUITS

$$v = L \frac{di}{dt}$$

or

$$V_{max} \sin \omega t = L \frac{di}{dt}$$

or

$$di = \frac{V_{max}}{L} \sin \omega t dt$$

Integrating both sides, we get

$$\int di = \int \frac{V_{max}}{L} \sin \omega t dt$$

or

$$i = \frac{V_{max}}{L} \left(-\frac{\cos \omega t}{\omega} \right) = \frac{V_{max}}{\omega L} \sin \left[\omega t - \frac{\pi}{2} \right]$$

or

$$i = \frac{V_{max}}{X_L} \sin \left[\omega t - \frac{\pi}{2} \right] \quad \dots(ii)$$

where $X_L = \omega L$ (opposition offered to the flow of alternating current by a pure inductances) and is called **Inductive reactance**. It is given in ohms if L is in henry and ω is in radian/second.

The value of current will be maximum when $\sin \left(\omega t - \frac{\pi}{2} \right) = 1$

$$\therefore I_{max} = \frac{V_{max}}{X_L}$$

Substituting this value in eqn. (ii), we get

$$i = I_{max} \sin \left(\omega t - \frac{\pi}{2} \right) \quad \dots(iii)$$

Power. Refer to Fig. 24 (c)

$$\begin{aligned} \text{Instantaneous power, } p &= vi = V_{max} \sin \omega t \times I_{max} \sin \left(\omega t - \frac{\pi}{2} \right) \\ &= -V_{max} I_{max} \sin \omega t \cdot \cos \omega t \\ &= -\frac{V_{max} I_{max}}{2} \times 2 \sin \omega t \cos \omega t \\ &= -\frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cdot \sin 2\omega t \end{aligned}$$

$$\therefore \text{ Power for the whole cycle, } P = -\frac{V_{max}}{\sqrt{2}} \frac{I_{max}}{\sqrt{2}} \int_0^{2\pi} \sin 2\omega t = 0$$

Hence *average power consumed in a pure inductive circuit is zero.*

Hence in a *pure inductive circuit*, we have :

1. Current $I = \frac{V}{X_L} = \frac{V}{\omega L} = \frac{V}{2\pi fL}$ amp.
2. Current always *lags* behind the voltage by 90° .
3. Average power consumed is *zero*.

Variation of X_L and f :

Since $X_L = \omega L = 2\pi fL$, and here if L is constant, then

$$X_L \propto f$$

Fig. 25, shows the variation. As frequency is increased X_L increases and the current taken by the circuit decreases.

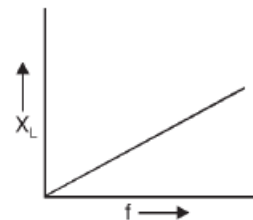
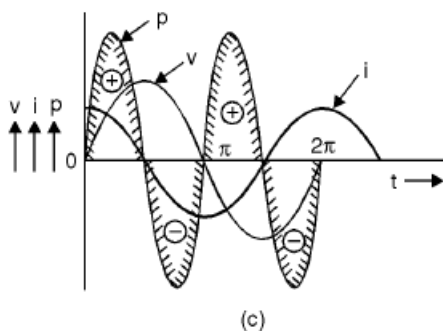
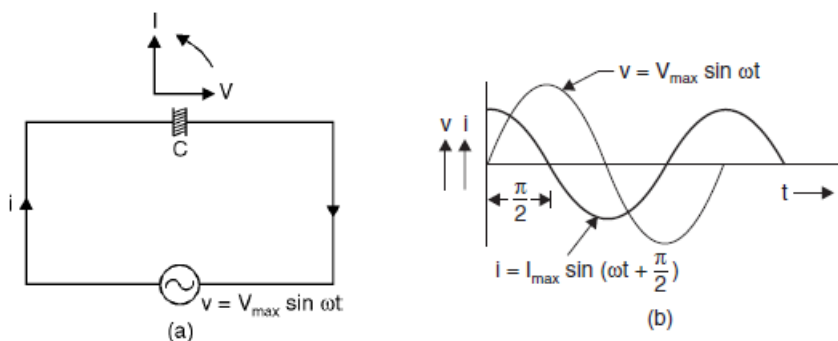


Fig. 25. Variation of X_L with f .

UNIT II AC CIRCUITS

4.3. A.C. Through Pure Capacitance Alone

The circuit containing a pure capacitor of capacitance C farad is shown in Fig. 26 (a). Let the alternating voltage applied across the circuit be given by the equation,



$$v = V_{max} \sin \omega t$$

Charge on the capacitor at any instant,

$$q = C v$$

Current through the circuit,

$$i = \frac{dq}{dt} = \frac{d}{dt} (C V_{max} \sin \omega t) = \omega C V_{max} \cos \omega t$$

or

$$i = \frac{V_{max}}{1/\omega C} \sin \left(\omega t + \frac{\pi}{2} \right)$$

$$\therefore i = \frac{V_{max}}{X_C} \sin \left(\omega t + \frac{\pi}{2} \right) \quad \dots(ii)$$

The denominator $X_C = \frac{1}{\omega C}$ (opposition offered to the flow of alternating current by a pure capacitor) is known as *capacitive reactance*.

It is given in ohms if C is in farad and ω in radian/second.

The value of current will be maximum when $\sin \left(\omega t + \frac{\pi}{2} \right) = 1$

$$\therefore I_{max} = \frac{V_{max}}{X_C}$$

Substituting this value in eqn. (ii), we get

$$i = I_{max} \sin \left(\omega t + \frac{\pi}{2} \right) \quad \dots(iii)$$

UNIT II AC CIRCUITS

Power. Refer to Fig. 26 (c)

Instantaneous power,

$$\begin{aligned}
 p = vi &= V_{max} \sin \omega t \times I_{max} \sin \left(\omega t + \frac{\pi}{2} \right) \\
 &= V_{max} I_{max} \sin \omega t \cos \omega t = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \sin 2\omega t
 \end{aligned}$$

$$\text{Power for the whole cycle} = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \int_0^{2\pi} \sin 2\omega t = 0$$

This fact is graphically illustrated in Fig. 26 (c). It may be noted that, during the first quarter cycle, what so ever power or energy is supplied by the source is stored in the electric field set-up between the capacitor plates. During the next quarter cycle, the electric field collapses and the power or energy stored in the field is returned to the source. The process is repeated in each alternation and this circuit does not absorb any power.

Hence in a *pure capacitive circuit*, we have

1. $I = \frac{V}{X_C} = V \times 2\pi fC$ amps.
2. Current always leads the applied voltage by 90° .
3. Power consumed is *zero*.

Variation of X_C and f :

Since $X_C = \frac{1}{2\pi fC}$ and if C is kept constant, than

$$X_C \propto \frac{1}{f}$$

Fig. 27 shows the variation. As the frequency increases X_C decreases, so the current increases.

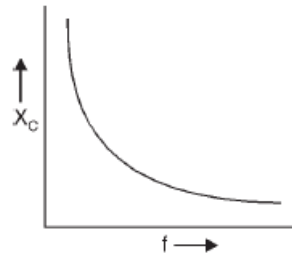


Fig. 27

4.4. Phasor Algebra

The following are the methods of representing vector quantities :

1. Symbolic notation
2. Trigonometrical form
3. Exponential form
4. Polar form.

A vector as shown in Fig. 28 may be described in the above forms as follows :

1. *Symbolic notation* :

$$E = a + jb$$

2. *Trigonometrical form* :

$$\begin{aligned}
 E &= \sqrt{a^2 + b^2} (\cos \theta + j \sin \theta) \\
 &= \sqrt{a^2 + b^2} (\cos \theta \pm j \sin \theta)
 \end{aligned}$$

3. *Exponential form* :

$$\begin{aligned}
 E &= \sqrt{a^2 + b^2} e^{+j\theta} \\
 &= \sqrt{a^2 + b^2} e^{\pm j\theta}
 \end{aligned}$$

4. *Polar form* :

$$\begin{aligned}
 E &= \sqrt{a^2 + b^2} \angle \theta \\
 &= \sqrt{a^2 + b^2} \angle \pm \theta
 \end{aligned}$$

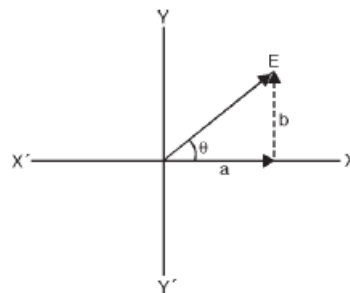


Fig. 28

.....in general

.....in general

.....in general

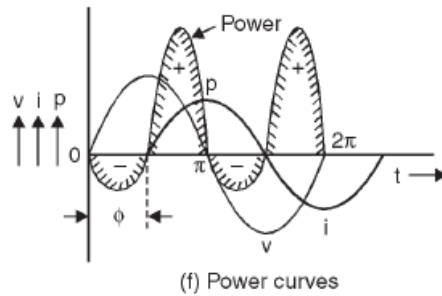
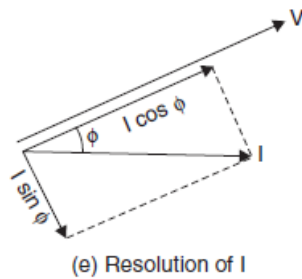
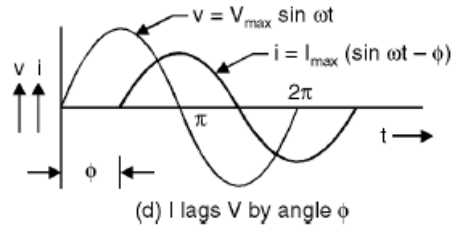
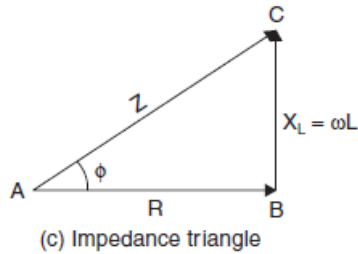
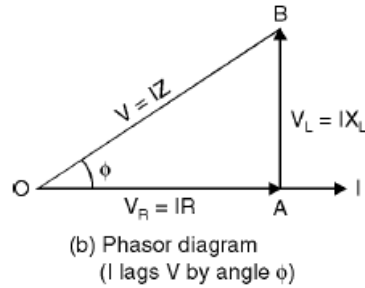
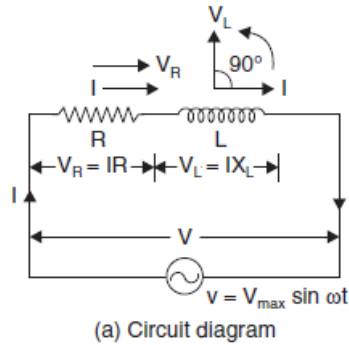
UNIT II AC CIRCUITS

4.5. A.C. Series Circuits

Under this heading we shall discuss *R-L*, *R-C* and *R-L-C* series circuits.

4.5.1. R-L circuit (Resistance and inductance in series)

Fig. 33 (a) shows a pure resistance *R* and a pure inductive coil of inductance *L* connected in series. Such a circuit is known as *R-L* circuit (usually met a cross in practice).



Let V = R.M.S. value of the applied voltage,
 I = R.M.S. value of the resultant current,
 $V_R = IR$ = Voltage drop across *R* (in phase with *I*), and
 $V_L = IX_L$ = Voltage drop across *L* (coil), ahead of *I* by 90° .

The voltage drop V_R and V_L and shown in voltage triangle *OAB* in Fig. 33 (b), *I* being taken as the reference vector in the phasor diagram. Vector *OA* represents ohmic drop V_R and *AB* represents inductive drop V_L . Vector *OB* represents the applied voltage *V* which is the vector sum of the two (i.e., V_R and V_L).

$$\therefore V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{R^2 + X_L^2}$$

or

$$I = \frac{V}{\sqrt{R^2 + (X_L)^2}} = \frac{V}{Z}$$

where $Z = \sqrt{R^2 + X_L^2}$ (total opposition offered to the flow of alternating current by *R-L* series circuit) is known as **impedance** of the circuit.

UNIT II

AC CIRCUITS

As seen from the "impedance triangle" ABC [Fig. 33 (c)],

$$Z^2 = R^2 + X_L^2$$

i.e., (Impedance)² = (Resistance)² + (Inductive reactance)²

From Fig. 33 (b) it is evident that voltage V leads the current by an angle ϕ such that,

$$\tan \phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{\text{Inductive reactance}}{\text{Resistance}}$$

$$\therefore \phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

The same is illustrated graphically in Fig. 33 (d).

In other words I lags V by an angle ϕ .

Power factor, $\cos \phi = \frac{R}{Z}$ [From Fig. 33 (c)]

Thus, if the applied voltage is given by $v = V_{\max} \sin \omega t$, then current equation is given as,

$$i = I_{\max} \sin (\omega t - \phi),$$

where

$$I_{\max} = \frac{V_{\max}}{Z}$$

In the Fig. 33 (e), I has been shown resolved into two components, $I \cos \phi$ along V and $I \sin \phi$ in quadrature (i.e., perpendicular) with V .

Mean power consumed by the circuit

$$= V \times I \cos \phi \text{ (i.e., component of } I \text{ which is in phase with } V)$$

i.e.,

$$P = VI \cos \phi \text{ (= r.m.s. voltage } \times \text{ r.m.s. current } \times \cos \phi)$$

The term ' $\cos \phi$ ' is called the *power factor* $\left(= \frac{R}{Z} \right)$ of the circuit

It may be noted that :

— In A.C. circuit the product of r.m.s. volts and r.m.s. amperes gives volt-amperes (i.e., VA) and *not true power in watts*. True power (W) = volt-amperes (VA) \times power factor

or

$$\text{Watts} = \text{VA (Apparent power)} \times \cos \phi$$

— The power consumed is due to ohmic resistance only since pure inductance consumes no power.

$$\text{i.e.} \quad P = VI \cos \phi = VI \times \frac{R}{Z} = \frac{V}{Z} IR = I \times IR = I^2 R, \text{ watts}$$

$$(\because \cos \phi = R/Z \text{ and } \frac{V}{Z} = I)$$

This shows that power is actually consumed in *resistance only*; the inductor does not consume any power.

The power consumed in R - L circuit is shown graphically in Fig. 33 (f).

Thus in R - L circuit we have :

1. Impedance, $Z = \sqrt{R^2 + X_L^2}$ (where $X_L = \omega L = 2\pi \times fL$)

2. Current, $I = \frac{V}{Z}$

3. Power factor, $\cos \phi = \frac{R}{Z} \left(= \frac{\text{True power}}{\text{Apparent power}} = \frac{W}{\text{VA}} \right)$

[or angle of lag, $\phi = \cos^{-1} (R/Z)$]

4. Power consumed, $P = VI \cos \phi \left(= IZ \times I \times \frac{R}{Z} = I^2 R \right)$

Symbolic Notation :

$$Z = R + jX_L$$

The numerical value of impedance vector = $\sqrt{R^2 + X_L^2}$

The phase angle with the reference axis, $\phi = \tan^{-1} (X_L/R)$.

In polar form : $\bar{Z} = Z \angle \phi^\circ$.

UNIT II AC CIRCUITS

Apparent, Active (True or real) and Reactive Power :

Every circuit current has two components : (i) Active component and (ii) Reactive component.

“Active component” consumes power in the circuit while “reactive component” is responsible for the field which lags or leads the main current from the voltage.

In Fig. 34. active component is $I_{\text{active}} = I \cos \phi$, and reactive component is $I_{\text{reactive}} = I \sin \phi$

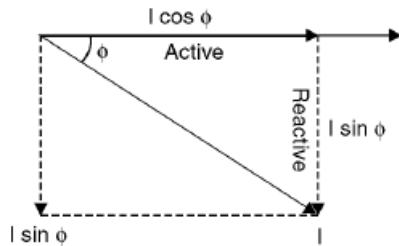


Fig. 34. Active and reactive components of circuit current I.

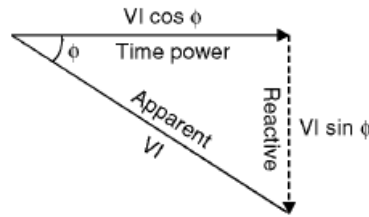


Fig. 35. Apparent, true and reactive power.

So,
$$I = \sqrt{(I_{\text{active}})^2 + (I_{\text{reactive}})^2}$$

Refer to Fig. 35.

(i) **Apparent power (S).** It is given by the product of r.m.s. values of applied voltage and circuit current.

$$\therefore S = VI = (I \times Z) \cdot I = I^2 Z \text{ volt-amperes (VA)}$$

(ii) **Active or true or real power (P or W).** It is the power which is actually dissipated in the circuit resistance.

$$P = I^2 R = VI \cos \phi \text{ watts}$$

(iii) **Reactive power (Q).** A pure inductor and a pure capacitor do not consume any power, since in a half cycle what so ever power is received from the source by these components the same is returned to the source. This power which flows back and forth (i.e., in both directions in the circuit) or reacts upon itself is called “reactive power.”

It may be noted that the current in phase with the voltage produces active or true or real power while the current 90° out of phase with the voltage contributes to reactive power.

In a R-L circuit, reactive power which is the power developed in the inductive reactance of the circuit, is given as :

$$\begin{aligned} Q &= I^2 X_L = I^2 Z \sin \phi = I \cdot (IZ) \sin \phi \\ &= VI \sin \phi \text{ volt-amperes-reactive (VAR)} \end{aligned}$$

Relation between VA, W and VAR

$$W = VA \cos \phi \quad \dots(i)$$

$$VAR = VA \sin \phi \quad \dots(ii)$$

$$\therefore VA = \frac{W}{\cos \phi} \quad \dots[\text{From (i)}]$$

and,
$$VA = \frac{VAR}{\sin \phi} \quad \dots[\text{From (ii)}]$$

$$\text{Power factor (p.f.)} = \frac{W}{VA} = \frac{\text{True power}}{\text{Apparent power}}$$

The larger bigger units of apparent, true and reactive power are kVA (or MVA), kW(or MW) and kVAR (or MVAR) respectively.

The power factor depends on the reactive power component. If it is made equal to the active power component, the power factor becomes unity.

UNIT II

AC CIRCUITS

The power factor depends on the reactive power component. If it is made equal to the active power component, the power factor becomes unity.

Example 20. A coil takes 2.5 amps. when connected across 200 volt 50 Hz mains. The power consumed by the coil is found to be 400 watts. Find the inductance and the power factor of the coil.

Solution. Current taken by the coil, $I = 2.5$ A

Applied voltage, $V = 200$ volts

Power consumed, $P = 400$ W

We know that $P = VI \cos \phi$

$$\text{or } 400 = 200 \times 2.5 \times \cos \phi \text{ or } \cos \phi = \frac{400}{200 \times 2.5} = 0.8$$

Hence power factor of coil is 0.8. (Ans.)

$$\text{Impedance of the coil, } Z = \frac{V}{I} = \frac{200}{2.5} = 80 \Omega$$

$$\text{Also } \frac{X_L}{Z} = \sin \phi$$

$$\therefore X_L = Z \sin \phi$$

$$= 80 \sin \phi = 80 \sqrt{1 - \cos^2 \phi}$$

$$= 80 \sqrt{1 - 0.8^2} = 80 \times 0.6 = 48 \Omega$$

But

$$X_L = 2\pi fL$$

\therefore

$$L = \frac{X_L}{2\pi f} = \frac{48}{2\pi \times 50} = 0.1529 \text{ H (henry)}. \text{ (Ans.)}$$

Example 21. A 100 V, 80 W lamp is to be operated on 230 volts, 50 Hz A.C. supply. Calculate the inductance of the choke required to be connected in series with lamp for its operation. The lamp can be taken as equivalent to a non inductive resistance.

Solution. Current through the lamp when connected across 100 V supply,

$$I = \frac{W}{V} = \frac{80}{100} = 0.8 \text{ A}$$

$$\text{Resistance of the lamp, } R = \frac{V}{I} = \frac{100}{0.8} = 125 \Omega$$

If a choke of inductance L henry is connected in series with the lamp to operate it on 230 V, the current through the choke will also be 0.8 A.

The impedance of the circuit when choke is connected in series with the lamp,

$$Z = \frac{V}{I} = \frac{230}{0.8} = 287.5 \Omega$$

$$\text{Reactance of choke coil, } X_L = \sqrt{Z^2 - R^2} = \sqrt{287.5^2 - 125^2} = 258.5 \Omega$$

But

$$X_L = 2\pi fL$$

or

$$L = \frac{X_L}{2\pi f} = \frac{258.5}{2\pi \times 50} = 0.825 \text{ H}$$

Hence inductance of choke coil, $L = 0.825$ H. (Ans.)

UNIT II AC CIRCUITS

4.5.2. R-C circuit (Resistance and capacitance in series)

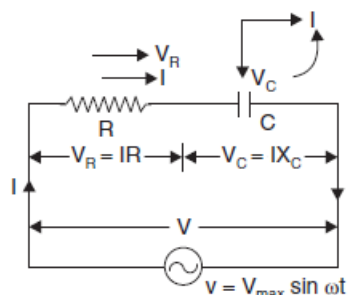
Fig. 41 (a) shows a pure resistance R (ohms) and a pure capacitor of capacitance C (farads) connected in series. Such a circuit is known as R - C circuit.

Let, V = R.M.S. value of the applied voltage,

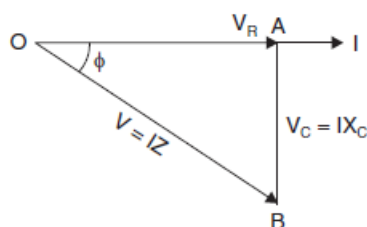
I = R.M.S. value of the resultant current,

$V_R = IR$ = Voltage drop across R (in phase with I), and

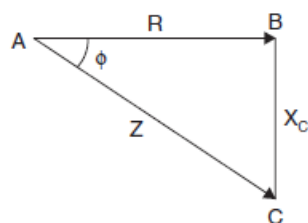
$V_C = IX_C$ = Voltage drop across C , lagging I by 90° .



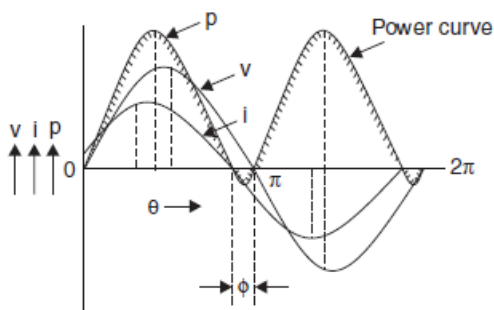
(a) Circuit diagram



(b) Phasor diagram
(I leads V by angle ϕ)



(c) Impedance triangle



(d) Power curve

Voltage drops V_R and V_C are shown in voltage triangle OAB in Fig. 41 (b) I being taken as the reference vector in the phasor diagram. Vector OA represents ohmic drop V_R and AB represents the capacitive drop V_C . Vector OB represents the applied voltage V , which is the vector sum of the V_R and V_C

$$\therefore V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = I \sqrt{R^2 + X_C^2}$$

or

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

UNIT II AC CIRCUITS

where $Z = \sqrt{R^2 + X_C^2}$ (total opposition offered to the flow of alternating current by R - C series circuit) is known as the **impedance** of the circuit.

As seen from the "impedance triangle" ABC [Fig. 41 (c)],

$$Z^2 = R^2 + X_C^2$$

i.e., (Impedance)² = (Resistance)² + (Capacitive reactance)²

From Fig. 41 (b) it is evident that I leads the voltage V by an angle ϕ such that,

$$\tan \phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{(1/\omega C)}{R} = \frac{\text{Capacitive reactance}}{\text{Resistance}}$$

$$\therefore \phi = \tan^{-1} \left(\frac{X_C}{R} \right)$$

The same is illustrated graphically in Fig. 41 (d).

In other words I leads V_R by an angle ϕ .

Power factor, $\cos \phi = \frac{R}{Z}$ [From Fig. 41 (c)]

Power. Refer to Fig. 41 (d),

Instantaneous power, $p = vi = V_{max} \sin \omega t \times I_{max} \sin (\omega t + \phi)$
 $= \frac{V_{max} I_{max}}{2} \times 2 \sin (\omega t + \phi) \sin \omega t$
 $= \frac{V_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} [\cos \phi - \cos (2\omega t + \phi)]$

Average power consumed in the circuit over a complete cycle,

$$P = \text{Average of } \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos \phi - \text{Average of } \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos (2\omega t + \phi)$$

or $P = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}} \cos \phi - \text{zero},$

or $P = V_{r.m.s.} \times I_{r.m.s.} \cos \phi = VI \cos \phi$

where $\cos \phi$ is the *power factor* of the circuit :

Alternatively, $P = VI \cos \phi = IZ \times I \times \frac{R}{Z} = I^2 R$

This shows that *power is actually consumed in resistance only ; the capacitor does not consume any power.*

Thus in R - C circuit, we have :

1. Impedance, $Z = \sqrt{R^2 + X_C^2}$ (where $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f}$, C being in farad)

2. Current, $I = \frac{V}{Z}$

3. Power factor, $\cos \phi = \frac{R}{Z}$ ($= \frac{\text{True power}}{\text{Apparent power}} = \frac{W}{VA}$)

[or angle of lead, $\phi = \cos^{-1} (R/Z)$]

4. Power consumed, $P = VI \cos \phi (= I^2 R)$.

UNIT II AC CIRCUITS

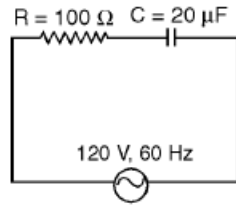
Example 29. A capacitance of $20 \mu\text{F}$ and a resistance of 100 ohms are connected in series across 120 V , 60 Hz mains. Determine the average power expended in the circuit.

Also draw the vector diagram.

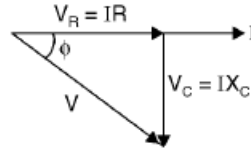
Solution.

$$R = 100 \Omega$$

$$C = 20 \mu\text{F} = 20 \times 10^{-6} \text{ F (farad)}$$



(a) R-C circuit



(b) Vector/phasor diagram

Fig. 42

Capacitive reactance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 20 \times 10^{-6}} = 159 \Omega$$

Impedance of the circuit,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + 159^2} = 188 \Omega$$

Current through the circuit

$$I = \frac{V}{Z} = \frac{120}{188} = 0.638 \text{ A}$$

Power factor,

$$\cos \phi = \frac{R}{Z} = \frac{100}{188} = 0.532$$

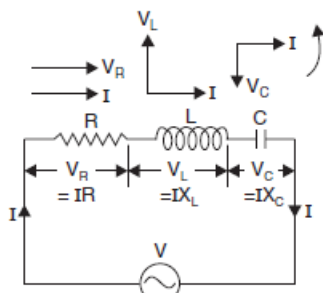
Average power expended in the circuit,

$$P_{av} = VI \cos \phi \\ = 120 \times 0.638 \times 0.532 = 40.75 \text{ W. (Ans.)}$$

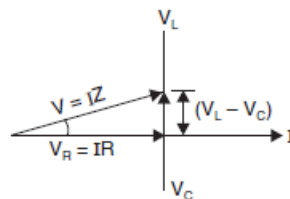
Fig. 42 (b) shows the vector/phasor diagram.

4.5.3. R-L-C circuit (Resistance, inductance and capacitance in series)

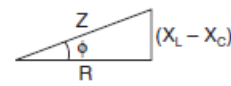
Fig. 45 shows a R-L-C circuit.



(a) R-L-C Circuit

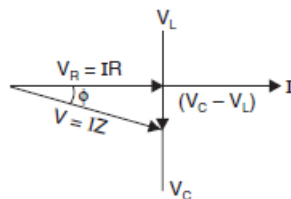


Phasor diagram

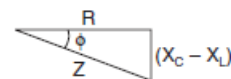


Impedance triangle

(b) $X_L > X_C$



Phasor diagram



Impedance triangle

(c) $X_C > X_L$

UNIT II AC CIRCUITS

Important formulae :

1. Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$
[where $X_L = 2\pi fL$, L in henry and $X_C = \frac{1}{2\pi fC}$, C in farad]
2. Current, $I = \frac{V}{Z}$
3. Power factor, $\cos \phi = \frac{R}{Z}$
[angle of lag (when $X_L > X_C$) or lead (when $X_C > X_L$), $\phi = \cos^{-1} \frac{R}{Z}$]
4. Power consumed $= VI \cos \phi (= I^2 R)$

4.6. A.C. Parallel Circuits

4.6.1. Introduction

Now-a-days, owing to multiple system of transmission and distribution, we come across parallel circuits (*i.e.*, impedances joined in parallel) more often. Practically all lighting and power circuits are constant voltage circuits with the loads connected in parallel. In a parallel A.C. circuit (like parallel D.C. circuit) the *voltage is the same across each branch*.

4.6.2. Methods for solving A.C. parallel circuits

The following *three* methods are available to solve such circuits :

1. Phasor or vector method
2. Admittance method
3. Vector algebra (symbolic method or *j*-method)

2. Admittance method

Admittance (denoted by symbol Y) of a circuit is defined as the *reciprocal of its impedance*.

$$\therefore Y = \frac{1}{Z} = \frac{I}{V} \quad \text{or} \quad Y = \frac{\text{r. m. s. amperes}}{\text{r. m. s. volts}}$$

The unit of admittance is *siemens* (S). The old unit was *mho* (Ω).

As the impedance Z of a circuit has two components R and X (See Fig. 55), similarly, shown in Fig. 56, admittance Y also has two components G (*conductance*- X -component) and B (*susceptance*- Y -component).

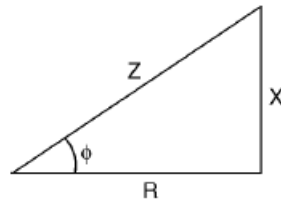


Fig. 55

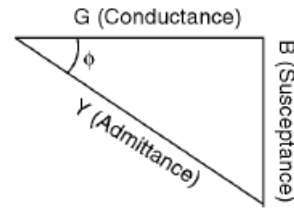


Fig. 56

Obviously, $G = Y \cos \phi = \frac{1}{Z} \cdot \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}$

Similarly, $B = Y \sin \phi = \frac{1}{Z} \cdot \frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2 + X^2}$

\therefore Admittance, $Y = \sqrt{G^2 + B^2}$ just as $Z = \sqrt{R^2 + X^2}$

The units of G , B and Y are in Siemens. Here, we shall consider capacitive susceptance as +ve and inductive capacitance as -ve.

UNIT II

AC CIRCUITS

Application of admittance method in solution of single-phase parallel circuits :

Refer to Fig. 57. Determine conductance and susceptance of individual branches from the relations

$$G = \frac{R}{Z^2} \text{ and } B = \frac{X}{Z^2}$$

Taking B as +ve if X is capacitive and as -ve if X is inductive. Let the conductances of the three branches of circuit shown in Fig. 57 be G_1 , G_2 and G_3 respectively and susceptances be B_1 , B_2 and B_3 respectively. Total conductance is found by merely adding the conductances of three branches. Similarly, total susceptance is found by algebraically adding the individual susceptances of different branches.

$$\therefore \text{ Total conductance, } G = G_1 + G_2 + G_3$$

$$\text{and, total susceptance, } B = B_1 + B_2 + B_3$$

$$\therefore \text{ Total admittance } Y = \sqrt{G^2 + B^2} \quad \dots(10)$$

$$\text{Total current, } I = VY \quad \dots(11)$$

$$\text{Power factor, } \cos \phi = \frac{G}{Y} \quad \dots(12)$$

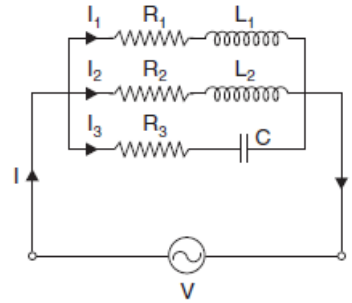


Fig. 57. Admittance method.

UNIT III

INTRODUCTION TO ELECTRICAL MECHANICS

1.DC GENERATOR - INTRODUCTION

An electrical generator is a device that **converts mechanical energy to electrical energy**, generally using electromagnetic induction. The source of mechanical energy may be a reciprocating or turbine steam engine, water falling through a turbine or waterwheel, an internal combustion engine, a wind turbine, a hand crank, or any other source of mechanical energy.

The Dynamo was the first electrical generator capable of delivering power for industry. The dynamo uses electromagnetic principles to convert mechanical rotation into an alternating electric current. A dynamo machine consists of a stationary structure which generates a strong magnetic field, and a set of rotating windings which turn within that field. On small machines the magnetic field may be provided by a permanent magnet; larger machines have the magnetic field created by electromagnets. The energy conversion in generator is based on the principle of the production of dynamically induced e.m.f. whenever a conductor cuts magnetic flux, dynamically induced e.m.f is produced in it according to Faraday's Laws of Electromagnetic induction. This e.m.f causes a current to flow if the conductor circuit is closed.

2. CONSTRUCTION OF D.C. MACHINES

A D.C. machine consists mainly of two parts: the stationary part called stator and the rotating part called rotor. The stator consists of main poles used to produce magnetic flux, commutating poles or interpoles in between the main poles to avoid sparking at the commutator but in the case of small machines sometimes the interpoles are avoided and finally the frame or yoke which forms the supporting structure of the machine. The rotor consists of an armature, a cylindrical metallic body or core with slots in it to place armature windings or bars, a commutator and brush gears. The magnetic flux path in a motor or generator is shown below and it is called the magnetic structure of generator or motor.

The major parts can be identified as

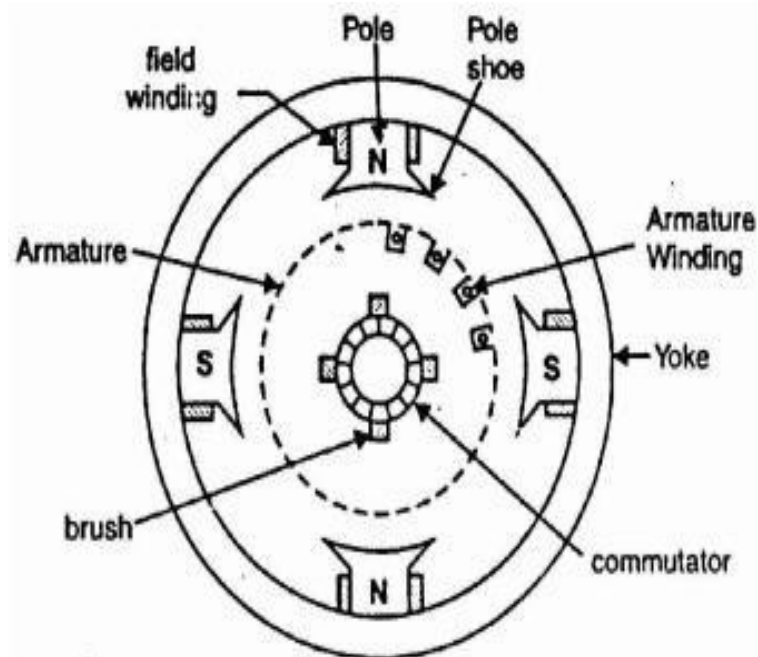
- (i) Frame
- (ii) Poles
- (iii) Armature
- (iv) Field winding
- (v) Commutator
- (vi) Brush
- (vii) Other mechanical parts

(i).Frame

Frame is the stationary part of a machine on which the main poles and commutator poles are bolted and it forms the supporting structure by connecting the frame to the bed plate. The ring-shaped body portion of the frame which makes the magnetic path for the magnetic fluxes from the main poles and interpoles is called Yoke.

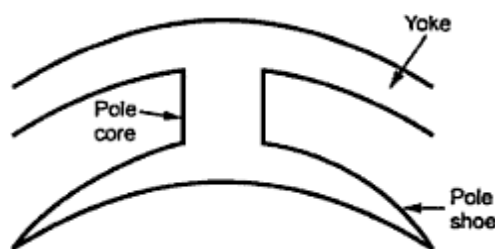
Why we use cast steel instead of cast iron for the construction of Yoke?

In early days Yoke was made up of cast iron but now it is replaced by cast steel. This is because cast iron is saturated by a flux density of 0.8 Wb/sq.m where as saturation with cast iron steel is about 1.5 Wb/sq.m. So for the same magnetic flux density the cross section area needed for cast steel is less than cast iron hence the weight of the machine too. If we use cast iron there may be chances of blow holes in it while casting. So now rolled steels are developed and these have consistent magnetic and mechanical properties.



(ii) Poles:

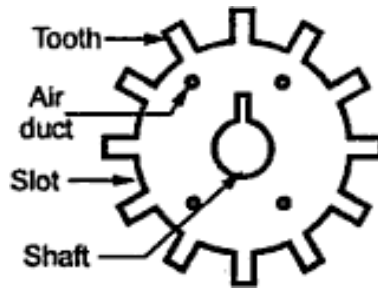
Solid poles of fabricated steel with separate/integral pole shoes are fastened to the frame by means of bolts. Pole shoes are generally laminated. Sometimes pole body and pole shoe are formed from the same laminations. The pole shoes are shaped so as to have a slightly increased air gap at the tips. Inter-poles are small additional poles located in between the main poles.



These can be solid, or laminated just as the main poles. These are also fastened to the yoke by bolts. Sometimes the yoke may be slotted to receive these poles. The inter poles could be of tapered section or of uniform cross section. These are also called as commutating poles or com poles. The width of the tip of the com pole can be about a rotor slot pitch.

(iii) Armature

The armature is where the moving conductors are located. The armature is constructed by stacking laminated sheets of silicon steel. Thickness of these lamination is kept low to reduce eddy current losses. As the laminations carry alternating flux the choice of suitable material, insulation coating on the laminations, stacking it etc are to be done more carefully. The core is divided into packets to facilitate ventilation. The winding cannot be placed on the surface of the rotor due to the mechanical forces coming on the same. Open parallel sided equally spaced slots are normally punched in the rotor laminations.



These slots house the armature winding. Large sized machines employ a spider on which the laminations are stacked in segments. End plates are suitably shaped so as to serve as 'Winding supporters'. Armature construction process must ensure provision of sufficient axial and radial ducts to facilitate easy removal of heat from the armature winding.

(iv) Field windings:

In the case of wound field machines (as against permanent magnet excited machines) the field winding takes the form of a concentric coil wound around the main poles. These carry the excitation current and produce the main field in the machine. Thus the poles are created electromagnetically. Two types of windings are generally employed. In shunt winding large number of turns of small section copper conductor is used. The resistance of such winding would be an order of magnitude larger than the armature winding resistance.

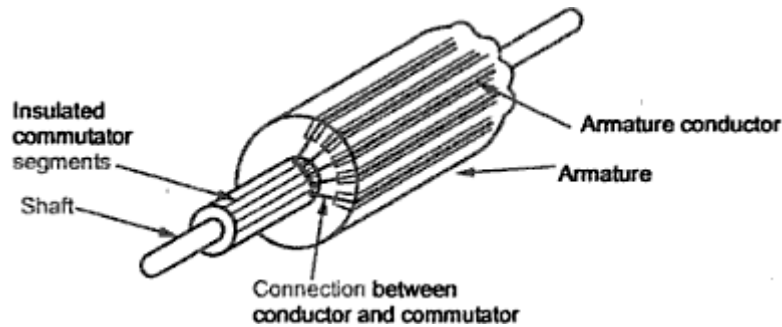
In the case of series winding a few turns of heavy cross section conductor is used. The resistance of such windings is low and is comparable to armature resistance. Some machines may have both the windings on the poles. The total ampere turns required to establish the necessary flux under the poles is calculated from the magnetic circuit calculations. The total mmf required is divided equally between north and south poles as the poles are produced in pairs. The mmf required to be shared between shunt and series windings are apportioned as per the design requirements. As these work on the same magnetic system they are in the form of concentric coils. Mmf 'per pole' is normally used in these calculations. Armature winding As mentioned earlier, if the armature coils are wound on the surface of the armature, such construction becomes mechanically weak.

The conductors may fly away when the armature starts rotating. Hence the armature windings are in general pre-formed, taped and lowered into the open slots on the armature. In the case of small machines, they can be hand wound. The coils are prevented from flying out due to the centrifugal forces by means of bands of steel wire on the surface of the rotor in small groves cut into it.

In the case of large machines slot wedges are additionally used to restrain the coils from flying away. The end portion of the windings are taped at the free end and bound to the winding carrier ring of the armature at the commutator end. The armature must be dynamically balanced to reduce the centrifugal forces at the operating speeds. Compensating winding One may find a bar winding housed in the slots on the pole shoes. This is mostly found in d.c. machines of very large rating. Such winding is called compensating winding. In smaller machines, they may be absent.

(v) Commutator:

Commutator is the key element which made the d.c. machine of the present day possible. It consists of copper segments tightly fastened together with mica/micanite insulating separators



on an insulated base. The whole commutator forms a rigid and solid assembly of insulated copper strips and can rotate at high speeds. Each commutator segment is provided with a 'riser' where the ends of the armature coils get connected. The surface of the commutator is machined and surface is made concentric with the shaft and the current collecting brushes rest on the same. Under-cutting the mica insulators that are between these commutator segments has to be done periodically to avoid fouling of the surface of the commutator by mica when the commutator gets worn out. Some details of the construction of the commutator are seen in Fig

(vi) Brush and brush holders:

Brushes rest on the surface of the commutator. Normally electro-graphite is used as brush material. The actual composition of the brush depends on the peripheral speed of the commutator and the working voltage. The hardness of the graphite brush is selected to be lower than that of the commutator. When the brush wears out the graphite works as a solid lubricant reducing frictional coefficient.

More number of relatively smaller width brushes are preferred in place of large broad brushes. The brush holders provide slots for the brushes to be placed. The connection Brush holder with a Brush and Positioning of the brush on the commutator from the brush is taken out by means of flexible pigtail. The brushes are kept pressed on the commutator with the help of springs. This is to ensure proper contact between the brushes and the commutator even under high speeds of operation. Jumping of brushes must be avoided to ensure arc free current collection and to keep the brushcontact drop low.

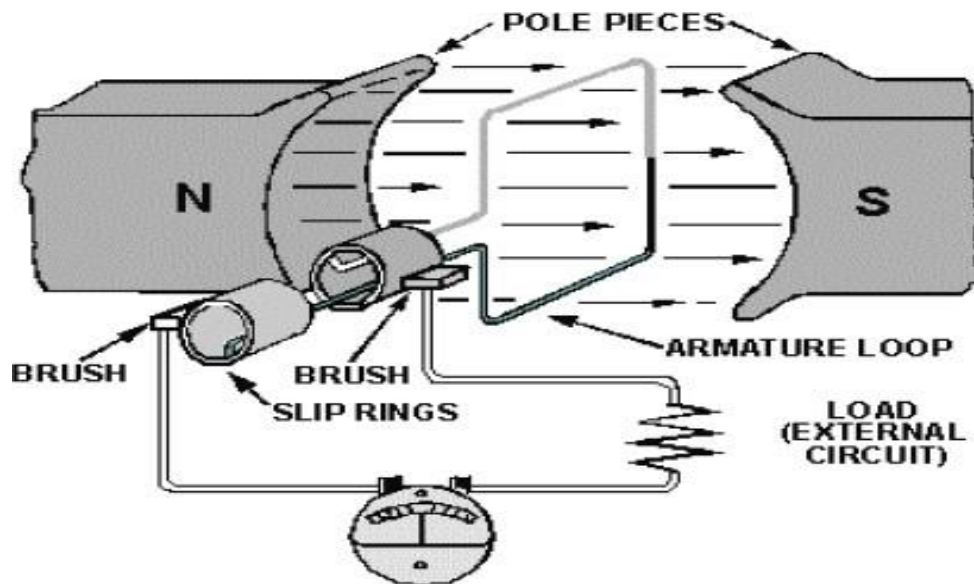
Other mechanical parts End covers, fan and shaft bearings form other important mechanical parts. End covers are completely solid or have opening for ventilation. They support the bearings which are on the shaft. Proper machining is to be ensured for easy assembly. Fans can be external or internal. In most machines the fan is on the non-commutator end sucking the air from the commutator end and throwing the same out. Adequate quantity of hot air removal has to be ensured. Bearings Small machines employ ball bearings at both ends. For larger machines roller bearings are used especially at the driving end. The bearings are mounted press-fit on the shaft. They are housed inside the end shield in such a manner that it is not necessary to remove the bearings from the shaft for dismantling.

(viii) End Shields or Bearings

If the armature diameter does not exceed 35 to 45 cm then in addition to poles end shields or frame head with bearing are attached to the frame. If the armature diameter is greater than 1 m pedestral type bearings are mounted on the machine bed plate outside the frame. These bearings could be ball or roller type but generally plain pedestral bearings are employed. If the diameter of the armature is large a brush holder yoke is generally fixed to the frame

3.PRINCIPLE OF OPERATION

DC generator converts mechanical energy into electrical energy. when a conductor move in a magnetic field in such a way conductors cuts across a magnetic flux of lines and emf produces in a generator and it is defined by faradays law of electromagnetic induction emf causes current to flow if the conductor circuit is closed.

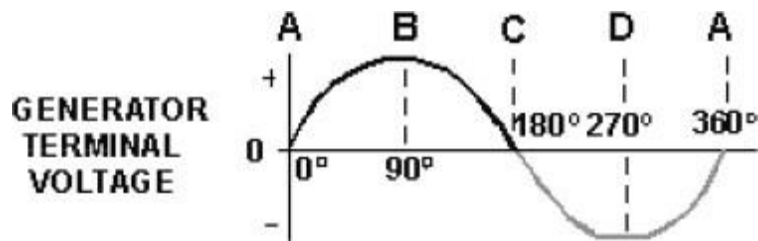
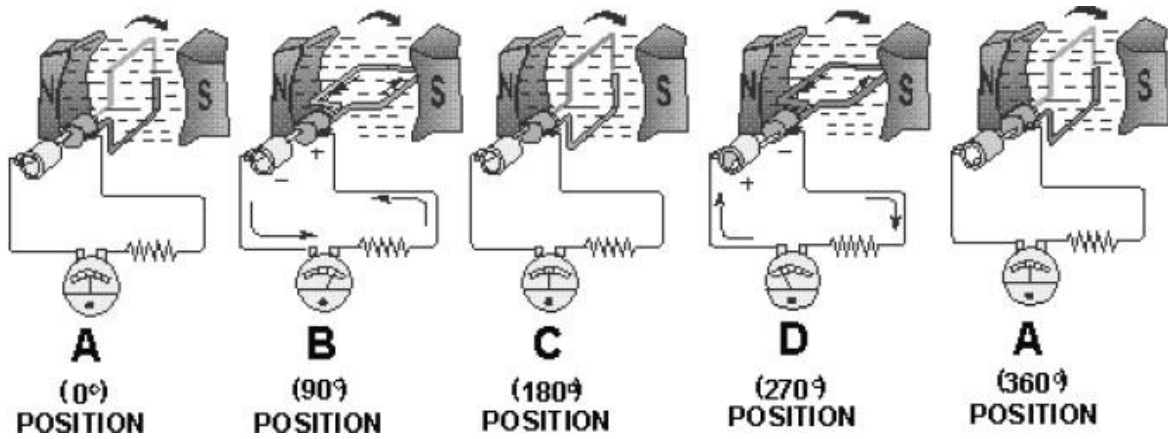


The pole pieces (marked N and S) provide the magnetic field. The pole pieces are shaped and positioned as shown to concentrate the magnetic field as close as possible to the wire loop. The loop of wire that rotates through the field is called the ARMATURE. The ends of the armature loop are connected to rings called SLIP RINGS. They rotate with the armature. The brushes, usually made of carbon, with wires attached to them, ride against the rings. The generated voltage appears across these brushes.

The elementary generator produces a voltage in the following manner (fig. 1-3). The armature loop is rotated in a clockwise direction. The initial or starting point is shown at position A. (This will be considered the zero-degree position.)

At 0° the armature loop is perpendicular to the magnetic field. The black and white conductors of the loop are moving parallel to the field. The instant the conductors are moving parallel to the magnetic field, they do not cut any lines of flux. Therefore, no emf is induced in the conductors, and the meter at position A indicates zero. This position is called the NEUTRAL PLANE. As the armature loop rotates from position A (0°) to position B (90°), the conductors cut through more and more lines of flux, at a continually increasing angle.

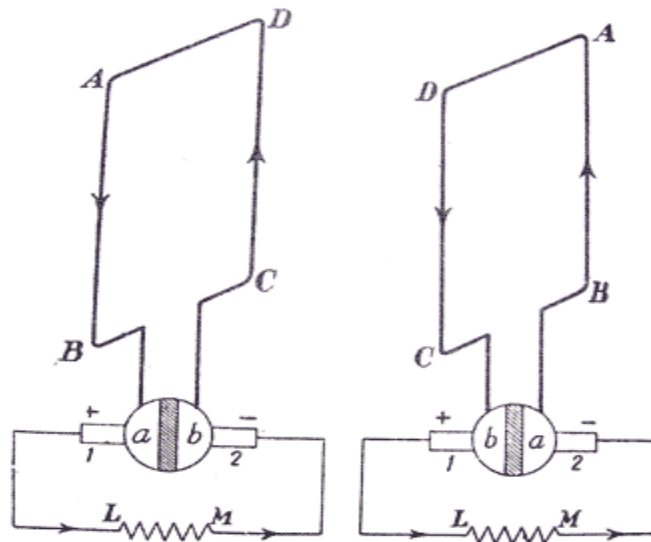
At 90° they are cutting through a maximum number of lines of flux and at maximum angle. The result is that between 0° and 90° , the induced emf in the conductors builds up from zero to a maximum value. Observe that from 0° to 90° , the black conductor cuts DOWN through the field. At the same time the white conductor cuts UP through the field



The induced emfs in the conductors are series-adding. This means the resultant voltage across the brushes (the terminal voltage) is the sum of the two induced voltages. The meter at position B reads maximum value. As the armature loop continues rotating from 90° (position B) to 180° (position C), the conductors which were cutting through a maximum number of lines of flux at position B now cut through fewer lines. They are again moving parallel to the magnetic field at position C. They no longer cut through any lines of flux.

As the armature rotates from 90° to 180° , the induced voltage will decrease to zero in the same manner that it increased during the rotation from 0° to 90° . The meter again reads zero. From 0° to 180° the conductors of the armature loop have been moving in the same direction through the magnetic field. Therefore, the polarity of the induced voltage has remained the same. This is shown by points A through C on the graph. As the loop rotates beyond 180° (position C), through 270° (position D), and back to the initial or starting point (position A), the direction of the cutting action of the conductors through the magnetic field reverses.

Now the black conductor cuts UP through the field while the white conductor cuts DOWN through the field. As a result, the polarity of the induced voltage reverses. Following the sequence shown by graph points C, D, and back to A, the voltage will be in the direction opposite to that shown from points A, B, and C. The terminal voltage will be the same as it was from A to C except that the polarity is reversed (as shown by the meter deflection at position D). The voltage output waveform for the complete revolution of the loop is shown on the graph in figure



4.GENERATOR E.M.F EQUATION

Let

Φ = flux/pole in weber

Z = total number of armature conductors
= No. of slots x No. of conductors/slot

P = No. of generator poles

A = No. of parallel paths in armature

N = armature rotation in revolutions per minute (r.p.m)

E = e.m.f induced in any parallel path in armature

E_g = e.m.f generated in any one of the parallel paths i.e

E . Average e.m.f generated /conductor = $d\Phi/dt$ volt ($n=1$)

Now, flux cut/conductor in one revolution $d\Phi = \Phi P$ Wb

No. of revolutions/second = $N/60$

Time for one revolution, $dt = 60/N$ second

Hence, according to Faraday's Laws of ElectroMagnetic Induction,

E.M.F generated/conductor is

$$\frac{d\Phi}{dt} = \frac{\Phi PN}{60}$$

For a simplex wave-wound generator

No. of parallel paths = 2

No. of conductors (in series) in one path = $Z/2$

E.M.F. generated/path is

$$\frac{\Phi PN}{60} \times \frac{Z}{2} = \frac{\Phi ZPN}{120} \text{ volt}$$

For a simplex lap-wound generator

No. of parallel paths = P

No. of conductors (in series) in one path = Z/P

E.M.F. generated/path

$$\frac{\Phi PN}{60} \times \frac{Z}{P} = \frac{\Phi ZN}{60} \text{ volt}$$

In general generated e.m.f

$$E_g = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volt}$$

where $A = 2$ for simplex wave-winding

$A = P$ for simplex lap-winding

5.DC MOTOR - INTRODUCTION

A machine that converts dc power into mechanical energy is known as dc motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of the force is given by Fleming's left hand rule.

How DC motors work?

There are different kinds of D.C. motors, but they all work on the same principles. When a permanent magnet is positioned around a loop of wire that is hooked up to a D.C. power source, we have the basics of a D.C. motor. In order to make the loop of wire spin, we have to connect a battery or DC power supply between its ends, and support it so it can spin about its axis. To allow the rotor to turn without twisting the wires, the ends of the wire loop are connected to a set of contacts called the commutator, which rubs against a set of conductors called the brushes. The brushes make electrical contact with the commutator as it spins, and are connected to the positive and negative leads of the power source, allowing electricity to flow through the loop. The electricity flowing through the loop creates a magnetic field that interacts with the magnetic field of the permanent magnet to make the loop spin.

6.PRINCIPLES OF OPERATION

It is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's Left-hand rule and whose magnitude is given by

$$\text{Force, } F = B I L \text{ newton}$$

Where

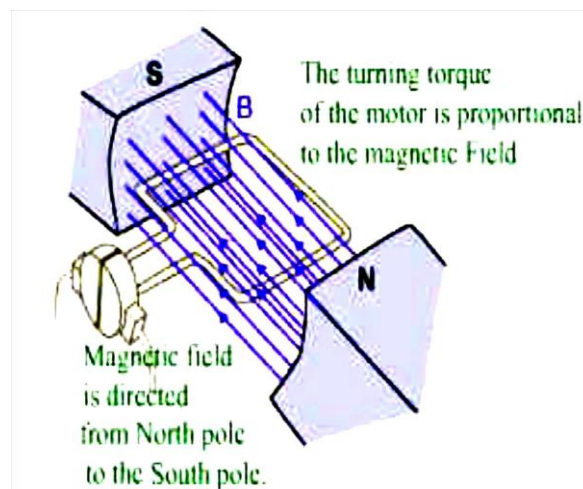
B is the magnetic field in weber/m².

I is the current in amperes and

L is the length of the coil in meter.

The force, current and the magnetic field are all in different directions.

If an Electric current flows through two copper wires that are between the poles of a magnet, an upward force will move one wire up and a downward force will move the other wire down.



BACK OR COUNTER EMF

When the armature of a d.c. motor rotates under the influence of the driving torque, the armature conductors move through the magnetic field and hence an e.m.f. is induced in them. The induced e.m.f. acts in opposite direction to the applied voltage V (Lenz's law) and is known as back or counter e.m.f. E_b .

7. SIGNIFICANCE OF BACK E.M.F

The presence of back e.m.f. makes the d.c. motor a self-regulating machine i.e., it makes the motor to draw as much armature current as is just sufficient to develop the torque required by the load. Back e.m.f. in a d.c. motor regulates the flow of armature current i.e., it automatically changes the armature current to meet the load requirement.

8. Torque equation of a DC motor

When armature conductors of a DC motor carry current in the presence of stator field flux, a mechanical torque is developed between the armature and the stator. Torque is given by the product of the force and the radius at which this force acts.

- Torque $T = F \times r$ (N-m) ... where, F = force and r = radius of the armature
- Work done by this force in once revolution = Force \times distance = $F \times 2\pi r$ (where, $2\pi r$ = circumference of the armature)
- Net power developed in the armature = work done / time
= (force \times circumference \times no. of revolutions) / time
= $(F \times 2\pi r \times N) / 60$ (Joules per second) eq. 2.1

But, $F \times r = T$ and $2\pi N/60 =$ angular velocity ω in radians per second. Putting these in the above equation 2.1

Net power developed in the armature = $P = T \times \omega$ (Joules per second)

Armature torque (T_a)

- The power developed in the armature can be given as, $P_a = T_a \times \omega = T_a \times 2\pi N/60$
- The mechanical power developed in the armature is converted from the electrical power, Therefore, mechanical power = electrical power
That means, $T_a \times 2\pi N/60 = E_b \cdot I_a$
- We know, $E_b = P\Phi NZ / 60A$
- Therefore, $T_a \times 2\pi N/60 = (P\Phi NZ / 60A) \times I_a$
- Rearranging the above equation,
 $T_a = (PZ / 2\pi A) \times \Phi \cdot I_a$ (N-m)

9. BRAKE TEST ON DC SHUNT MOTOR CHARACTERISTICS

Characteristics of DC Shunt Motor:

To study the performance of the DC shunt Motor various types of characteristics are to be studied.

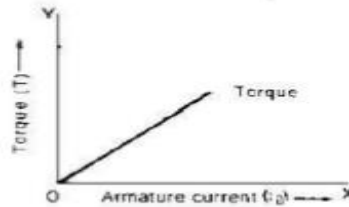
1. Torque Vs Armature current characteristics.
2. Speed Vs Armature current characteristics.
3. Speed Vs Torque characteristics.

Torque Vs Armature current characteristics of DC Shunt motor

This characteristic gives us information that, how torque of machine will vary with armature current, which depends upon load on the motor.

$$T \propto I_a$$

Thus,



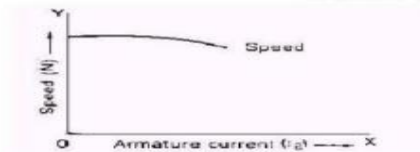
Speed Vs Armature current characteristics of DC Shunt Motor

The back emf of dc motor is $E_b = \frac{N\Phi ZP}{A60} = V - I_a R_a$

$$\text{Therefore } N = \frac{(V - I_a R_a) 60 A}{\Phi ZP} = \frac{K(V - I_a R_a)}{\Phi}$$

where $K = 60A/ZP$ and it is constant. In dc shunt motor, when supply voltage V is kept constant the shunt field current and hence flux per pole will also be constant.

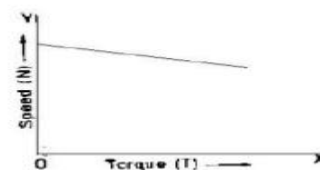
$$\therefore N \propto V - I_a R_a$$



- Therefore shunt motor is considered as constant speed motor.

Speed Vs Torque characteristics of DC Shunt motor

- From the above two characteristics of dc shunt motor, the torque developed and speed at various armature currents of dc shunt motor may be noted.
- If these values are plotted, the graph representing the variation of speed with torque developed is obtained.
- This curve resembles the speed Vs current characteristics as the torque is directly proportional to the armature current.



10. BRAKE TEST ON THE 3-PHASE INDUCTION MOTOR

AIM:

To conduct the brake test on the given 3-phase induction motor and plot its performance characteristics.

CIRCUIT DIAGRAM:

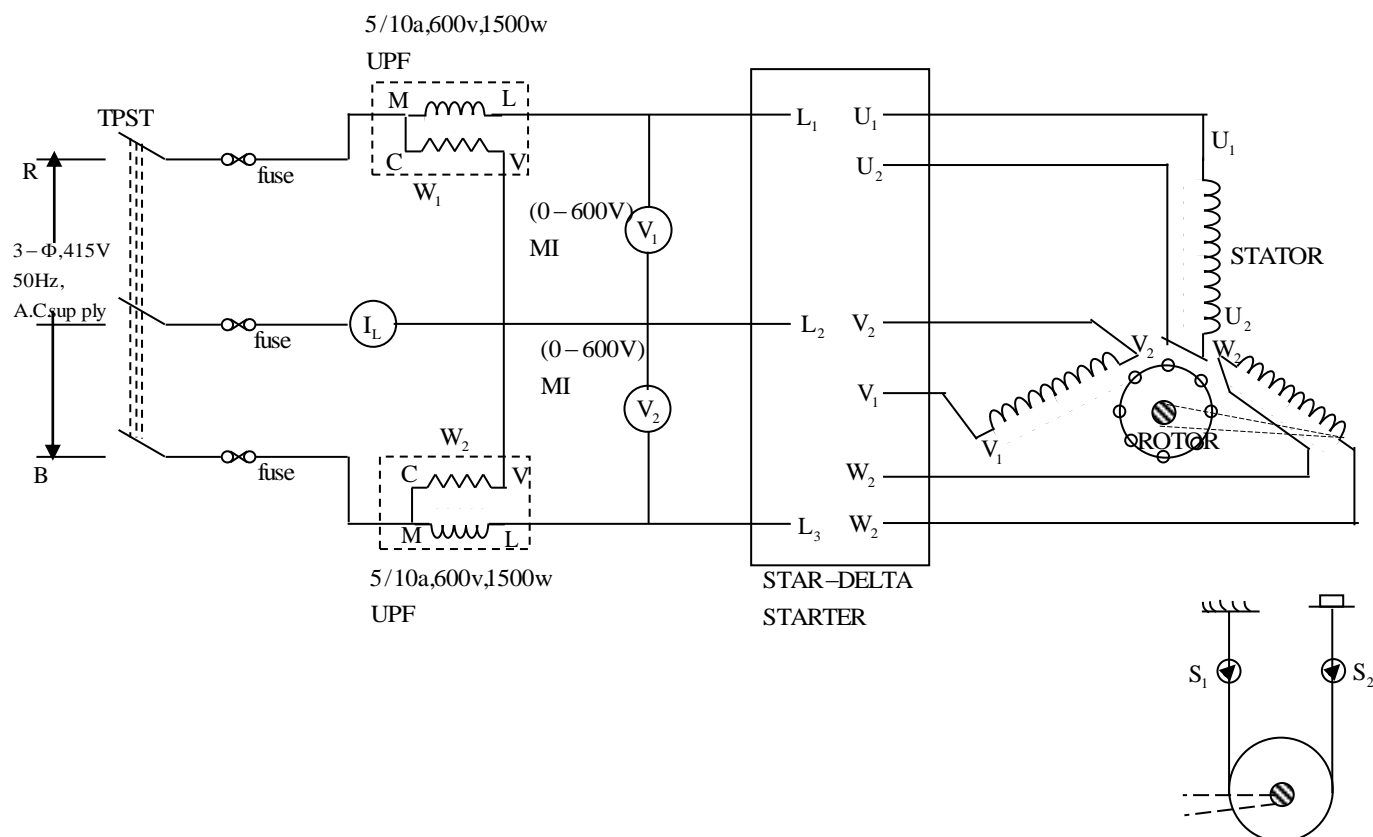


Fig 6

THEORY:

The brake test in a direct method of testing. It consists of applying a brake to a water-cooled pulley mounted on the shaft of the motor. A rope is wound round the pulley and its two ends are attached to two spring balances S_1 and S_2 . The tension of the rope can be adjusted with the help of swivels. Then,

The force acting tangentially on the pulley = $(S_1 - S_2)$ Kgs.

If R is the pulley radius, the torque at the pulley,

$$T_{sh} = (S_1 - S_2) R \text{ kg. Met.}$$

If " w " is the angular velocity of the motor.

$W = 2\pi N/60$, Where N is the speed in RPM.

$$\begin{aligned} \text{Motor output} &= T_{sh} \times w = 2\pi N(S_1 - S_2) \text{ kg.mt.wt} \\ &= 9.81 \times 2\pi N (s_1 - S_2) \text{ R watts.} \end{aligned}$$

The motor input can be measured directly as in the circuit diagram 6. For finding the performance characteristics, the speed of the motor can also be measured by a tachometer.

PROCEDURE:

1. Make the connections as per the circuit diagram in fig. 6.
2. Loosen the rope of the break drum such that $S_1 = S_2 = 0$.
3. Close the switch S and apply the rated 3-phase a.c. supply to the motor. Note the readings of all meters.
4. Gradually increase the load by tightening the rope and note down the readings of all meters and tabulate the results as shown below.

S.No.	S_1	S_2	W_1	W_2	I_L	Input P_I	Output	N	S=slip	T	η

5. Starting from no-load, take the readings as the line current is increased from $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 and $1\frac{1}{4}$ of its full value.
6. The output and input of the motors, Efficiency, Torque and slip can be calculated and the performance characteristic.
 - a) Load vs Efficiency
 - b) Load vs Speed
 - c) Load vs Torque
 - d) Load vs Slip and speed are drawn

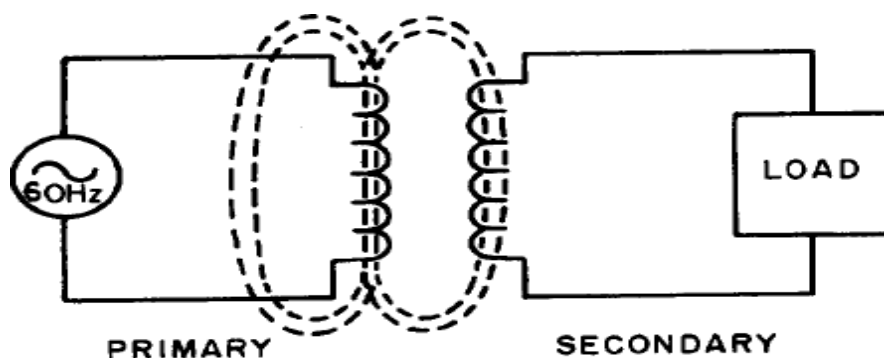
TRANSFORMER

11. BASIC OPERATION OF A TRANSFORMER

In its most basic form a transformer consists of:

- A primary coil or winding.
- A secondary coil or winding.
- A core that supports the coils or windings.

Refer to the transformer circuit in figure as you read the following explanation: The primary winding is connected to a 60 hertz ac voltage source. The magnetic field (flux) builds up (expands) and collapses (contracts) about the primary winding. The expanding and contracting magnetic field around the primary winding cuts the secondary winding and induces an alternating voltage into the winding. This voltage causes alternating current to flow through the load. The voltage may be stepped up or down depending on the design of the primary and secondary windings.



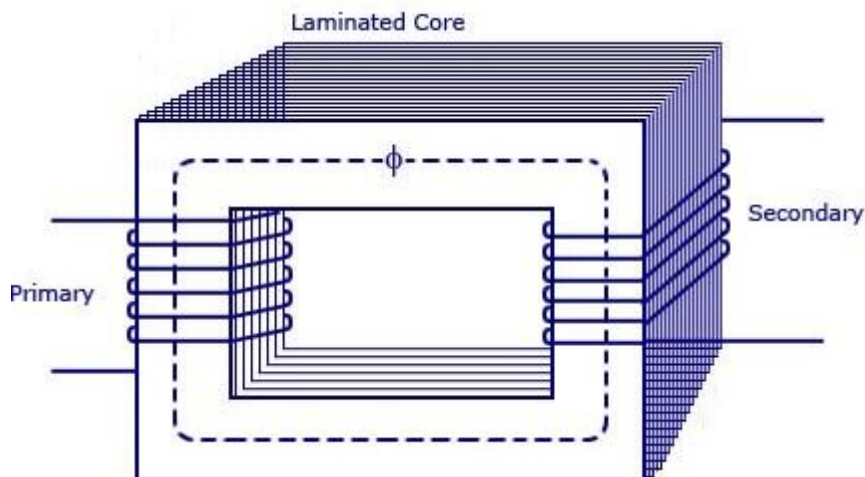
IDEAL TRANSFORMER

An ideal transformer is shown in the adjacent figure. Current passing through the primary coil creates a magnetic field. The primary and secondary coils are rapped around a core of very high magnetic permeability, such as iron, so that most of the magnetic flux passes through both the primary and secondary coils.

10.BASIC WORKING PRINCIPLE OF TRANSFORMER

A transformer can be defined as a static device which helps in the transformation of electric power in one circuit to electric power of the same frequency in another circuit. The voltage can be raised or lowered in a circuit, but with a proportional increase or decrease in the current ratings.

The main principle of operation of a transformer is mutual inductance between two circuits which is linked by a common magnetic flux. A basic transformer consists of two coils that are electrically separate and inductive, but are magnetically linked through a path of reluctance. The working principle of the transformer can be understood from the figure below.



As shown above the transformer has primary and secondary windings. The core laminations are joined in the form of strips in between the strips you can see that there are some narrow gaps right through the cross-section of the core. These staggered joints are said to be 'imbricated'. Both the coils have high mutual inductance. A mutual electro-motive force is induced in the transformer from the alternating flux that is set up in the laminated core, due to the coil that is connected to a source of alternating voltage. Most of the alternating flux developed by this coil is linked with the other coil and thus produces the mutual induced electro- motive force. The so produced electro-motive force can be explained with the help of Faraday's laws of Electromagnetic Induction as

$$e=M*dI/dt$$

If the second coil circuit is closed, a current flows in it and thus electrical energy is transferred magnetically from the first to the second coil.

The alternating current supply is given to the first coil and hence it can be called as the primary winding. The energy is drawn out from the second coil and thus can be called as the secondary winding.

In short, a transformer carries the operations shown below:

Transfer of electric power from one circuit to another. Transfer of electric power without any

change in frequency. Transfer with the principle of electromagnetic induction. The two electrical circuits are linked by mutual induction

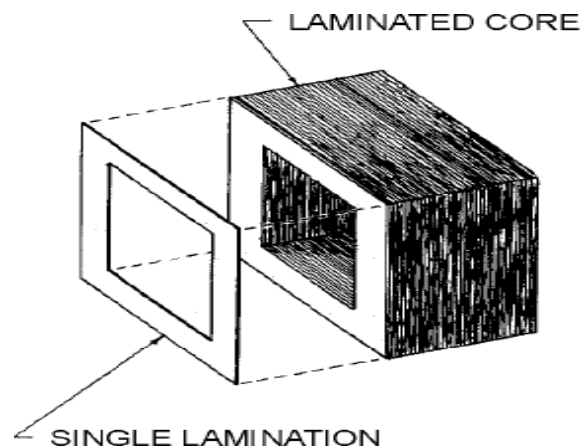
11. TRANSFORMER CONSTRUCTION

Two coils of wire (called windings) are wound on some type of core material. In some cases the coils of wire are wound on a cylindrical or rectangular cardboard form. In effect, the core material is air and the transformer is called an AIR-CORE TRANSFORMER. Transformers used at low frequencies, such as 60 hertz and 400 hertz, require a core of low-reluctance magnetic material, usually iron. This type of transformer is called an IRON-CORE TRANSFORMER. Most power transformers are of the iron-core type.

The principle parts of a transformer and their functions are:

- The CORE, which provides a path for the magnetic lines of flux.
- The PRIMARY WINDING, which receives energy from the ac source.
- The SECONDARY WINDING, which receives energy from the primary winding and delivers it to the load.
- The ENCLOSURE, which protects the above components from dirt, moisture, and mechanical damage.

(i) CORE



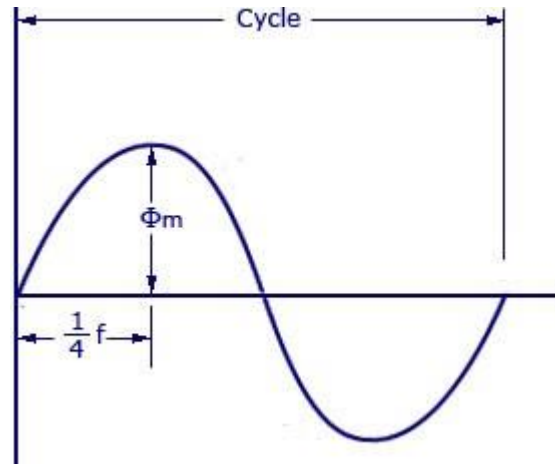
There are two main shapes of cores used in laminated-steel-core transformers. One is the HOLLOWCORE, so named because the core is shaped with a hollow square through the center. This shape of core. Notice that the core is made up of many laminations of steel it shows how the transformer windings are wrapped around both sides of the core.

(ii) WINDINGS

As stated above, the transformer consists of two coils called WINDINGS which are wrapped around a core. The transformer operates when a source of ac voltage is connected to one of the windings and a load device is connected to the other. The winding that is connected to the source is called the PRIMARY WINDING. The winding that is connected to the load is called the SECONDARY WINDING. The primary is wound in layers directly on a rectangular cardboard form.

12. EMF Equation of Transformer:

Let the applied voltage V_1 applied to the primary of a transformer, with secondary open-circuited, be sinusoidal (or sine wave). Then the current I_1 , due to applied voltage V_1 , will also be a sine wave. The mmf $N_1 I_1$ and core flux Φ will follow the variations of I_1 closely. That is the flux is in time phase with the current I_1 and varies sinusoidally.



Let,

N_A = Number of turns in primary

N_B = Number of turns in secondary

Φ_{max} = Maximum flux in the core in webers = $B_{max} \times A$

f = Frequency of alternating current input in hertz (Hz)

As shown in figure above, the core flux increases from its zero value to maximum value Φ_{max} in one quarter of the cycle, that is in $\frac{1}{4}$ frequency second.

Therefore, average rate of change of flux = $\Phi_{max} / \frac{1}{4} f = 4f \Phi_{max}$ Wb/s

Now, rate of change of flux per turn means induced electro motive force in volts.

Therefore,

average electro-motive force induced/turn = $4f \Phi_{max}$ volt

If flux Φ varies sinusoidally, then r.m.s value of induced e.m.f is obtained by multiplying the average value with form factor.

Form Factor = r.m.s. value/average value = 1.11 Therefore, r.m.s value of e.m.f/turn = $1.11 \times 4f \Phi_{max} = 4.44f \Phi_{max}$

Now, r.m.s value of induced e.m.f in the whole of primary winding

= (induced e.m.f./turn) X Number of primary turns

Therefore,

$E_A = 4.44f N_A \Phi_{max} = 4.44f N_A B_{max} A$

Similarly, r.m.s value of induced e.m.f in secondary is $E_B = 4.44f N_B \Phi_{max} = 4.44f N_B B_{max} A$

In an ideal transformer on no load, $V_A = E_A$ and $V_B = E_B$, where V_B is the terminal voltage

Voltage Transformation Ratio.

The ratio of secondary voltage to primary voltage is known as the voltage transformation ratio and is designated by letter K. i.e.

Voltage transformation ratio, $K = V_2/V_1 = E_2/E_1 = N_2/N_1$

Current Ratio.

The ratio of secondary current to primary current is known as current ratio and is reciprocal of voltage transformation ratio in an ideal transformer.

13.Three Phase Induction Motor- Introduction

The popularity of 3 phase induction motors on board ships is because of their simple, robust construction, and high reliability factor in the sea environment. A 3 phase induction motor can be used for different applications with various speed and load requirements. Electric motors can be found in almost every production process today. Getting the most out of your application is becoming more and more important in order to ensure cost-effective operations. The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a “transformer type” a.c. machine in which electrical energy is converted into mechanical energy.

Advantages

- (i) It has simple and rugged construction.
- (ii) It is relatively cheap.
- (iii) It requires little maintenance.
- (iv) It has high efficiency and reasonably good power factor.
- (v) It has self-starting torque.

Disadvantages

- (i) It is essentially a constant speed motor and its speed cannot be changed easily.
- (ii) Its starting torque is inferior to d.c. shunt motor.

14.Construction

The three phase induction motor is the most widely used electrical motor. Almost 80% of the mechanical power used by industries is provided by three phase induction motors because of its simple and rugged construction, low cost, good operating characteristics, absence of commutator and good speed regulation. In three phase induction motor the power is transferred from stator to rotor winding through induction. The Induction motor is also called asynchronous motor as it runs at a speed other than the synchronous speed. Like any other electrical motor induction motor also have two main parts namely rotor and stator. A 3-phase induction motor has two main parts (i) stator and (ii) rotor. The rotor

is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor. The main body of the Induction Motor comprises of two major parts as shows in Figure :

- i. Shaft for transmitting the torque to the load. This shaft is made up of steel.
- ii. Bearings for supporting the rotating shaft.
- iii. One of the problems with electrical motor is the production of heat during its rotation. In order to overcome this problem we need fan for cooling.
- iv. For receiving external electrical connection Terminal box is needed.
- v. There is a small distance between rotor and stator which usually varies from 0.4 mm to 4 mm. Such a distance is called air gap.

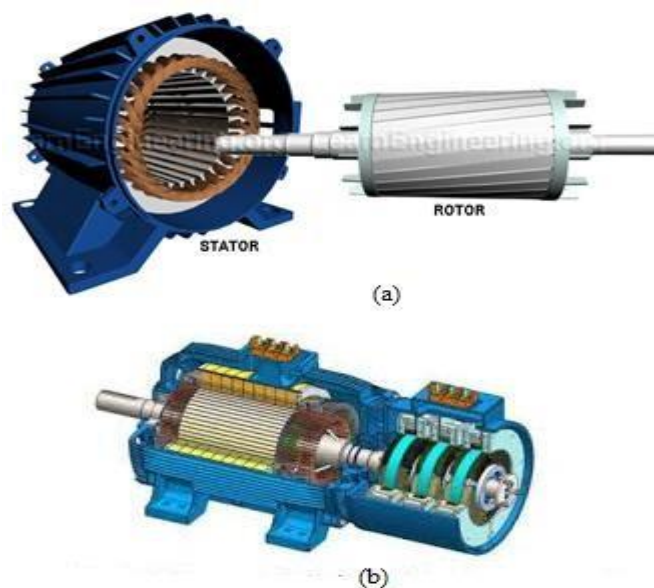


Fig. Three phase induction motor (a) squirrel cage rotor (b) slip ring rotor.

(1)Stator:

Stator: As its name indicates stator is a stationary part of induction motor. A stator winding is placed in the stator of induction motor and the three phase supply is given to it. Stator is made up of number of stampings in which different slots are cut to receive 3 phase winding circuit which is connected to 3 phase AC supply. The three phase windings are arranged in such a manner in the slots that they produce a rotating magnetic field after AC supply is given to them. The windings are wound for a definite number of poles depending upon the speed requirement, as speed is inversely proportional to the number of poles, given by the formula:

$$N_s = 120 f/p$$

Where

N_s = synchronous speed

f = Frequency

p = no. of poles



Figure Stator of three phase induction motor.

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations [See Fig.]. The insulated windings are connected to form a balanced 3-phase star or delta connected circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

Stator of Three Phase Induction Motor

The stator of the three phase induction motor consists of three main parts :

(i) Stator Frame:

It is the outer most part of the three phase induction motor. Its main function is to support the stator core and the field winding. It acts as a covering and it provides protection and mechanical strength to all the inner parts of the induction motor. The frame is either made up of die cast or fabricated steel. The frame of three phase induction motor should be very strong and rigid as the air gap length of motor is very small, otherwise rotor will not remain concentric with stator, which will give rise to unbalanced magnetic pull.

(ii) Stator Core:

The main function of the stator core is to carry the alternating flux. In order to reduce the eddy current loss, the stator core is laminated. These laminated types of structure are made up of stamping which is about 0.4 to 0.5 mm thick. All the stampings are stamped together to form stator core, which is then housed in stator frame. The stamping is generally made up of silicon steel, which helps to reduce the hysteresis loss occurring in motor.

(iii) Stator Winding or Field Winding:

The slots on the periphery of stator core of the motor carry three phase windings. These three

phase winding is supplied by three phase ac supply. The three phases of the winding are connected either in star or delta depending upon which type of starting method is used. The squirrel cage motor is mostly started by star – delta stator and hence the stator of squirrel cage motor is delta connected. The slip ring three phase induction motor are started by inserting resistances so, the stator winding of slip ring induction can be connected either in star or delta. The winding wound on the stator of three phase induction motor is also called field winding and when this winding is excited by three phase ac supply it produces a rotating magnetic.

(2)Rotor:

The rotor is a rotating part of induction motor. The rotor is connected to the mechanical load through the shaft. Rotor consists of cylindrical laminated core with parallel slots that carry conductor bars. Conductors are heavy copper or aluminium bars which fits in each slots. These conductors are brazed to the short circuiting end rings. The slots are not exactly made parallel to the axis of the shaft but are slotted a little skewed for the following reason, They reduces magnetic hum or noise and They avoid stalling of motor. The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types: Squirrel cage type and Wound type

Squirrel cage three phase induction motor:

The rotor of the squirrel cage three phase induction motor is cylindrical in shape and have slots on its periphery. The slots are not made parallel to each other but are bit skewed (skewing is not shown in the figure of squirrel cage rotor beside) as the skewing prevents magnetic locking of stator and rotor teeth and makes the working of motor more smooth and quieter. The squirrel cage rotor consists of aluminum, brass or copper bars. These aluminum, brass or copper bars are called rotor conductors and are placed in the slots on the periphery of the rotor. The rotor conductors are permanently shorted by the copper or aluminum rings called the end rings. In order to provide mechanical strength these rotor conductor are braced to the end ring and hence form a complete closed circuit resembling like a cage and hence got its name as “squirrel cage induction motor”. The squirrel cage rotor winding is made symmetrical. As the bars are permanently shorted by end rings, the rotor resistance is very small and it is not possible to add external resistance as the bars are permanently shorted. The absence of slip ring and brushes make the construction of Squirrel cage three phase

induction motor very simple and robust and hence widely used three phase induction motor. These motors have the advantage of adapting any number of pole pairs. The below diagram shows squirrel cage induction rotor having aluminum bars short circuit by aluminum end rings. It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. (8.3)]. This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator. Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.



Fig. squirrel cage rotor.

Advantages of squirrel cage induction rotor

- i. Its construction is very simple and rugged.
- ii. As there are no brushes and slip ring, these motors requires less maintenance.

Applications:

Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc.

Slip ring or wound three phase induction motor : In this type of three phase induction motor the rotor is wound for the same number of poles as that of stator but it has less number of slots and has less turns per phase of a heavier conductor. The rotor also carries star or delta winding similar to that of stator winding. The rotor consists of numbers of slots and rotor winding are

placed inside these slots. The three end terminals are connected together to form star connection. As its name indicates three phase slip ring induction motor consists of slip rings connected on same shaft as that of rotor. The three ends of three phase windings are permanently connected to these slip rings. The external resistance can be easily connected through the brushes and slip rings and hence used for speed control and improving the starting torque of three phase induction motor. The brushes are used to carry current to and from the rotor winding. These brushes are further connected to three phase star connected resistances. At starting, the resistance are connected in rotor circuit and is gradually cut out as the rotor pick up its speed. When the motor is running the slip ring are shorted by connecting a metal collar, which connect all slip ring together and the brushes are also removed. This reduces wear and tear of the brushes. Due to presence of slip rings and brushes the rotor construction becomes somewhat complicated therefore it is less used as compare to squirrel cage induction motor. It consists of a laminated cylindrical core and carries a 3- phase winding, similar to the one on the stator [See Fig.]. The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig. (). At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

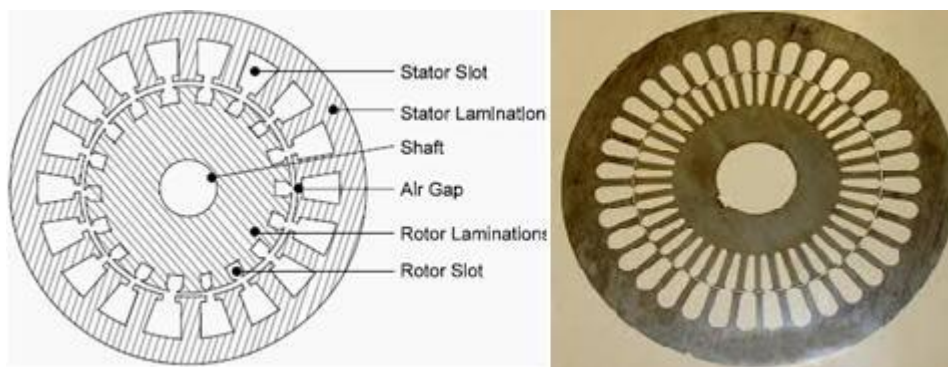


Fig Lamination of stator and rotor.

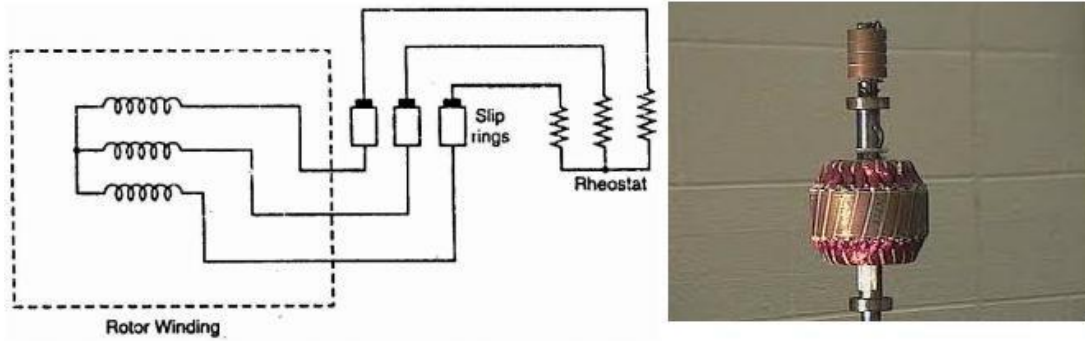


Figure Slip ring rotor.

Advantages of slip ring induction motor -

- A. It has high starting torque and low starting current.
- B. Possibility of adding additional resistance to control speed.

Application:

Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc.

Difference between Slip Ring and Squirrel Cage Induction Motor

SLIP RING OR PHASE WOUND	SQUIRREL CAGE
Construction is complicated due to presence of slip ring and brushes	Construction is very simple
The rotor winding is similar to the stator winding	The rotor consists of rotor bars which are permanently shorted with the help of end rings
We can easily add rotor resistance by using slip ring and brushes	Since the rotor bars are permanently shorted, its not possible to add external resistance
Due to presence of external resistance high starting torque can be obtained	Staring torque is low and cannot be improved

Slip ring and brushes are present	Slip ring and brushes are absent
Frequent maintenance is required due to presence of brushes	Less maintenance is required
The construction is complicated and the presence of brushes and slip ring makes the motor more costly	The construction is simple and robust and it is cheap as compared to slip ring induction motor
This motor is rarely used only 10 % industry uses slip ring induction motor	Due to its simple construction and low cost. The squirrel cage induction motor is widely used
Rotor copper losses are high and hence less efficiency	Less rotor copper losses and hence high efficiency
Speed control by rotor resistance method is possible	Speed control by rotor resistance method is not possible
Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc	Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc

15.Operation Principle

Unlike toys and flashlights, most homes, offices, factories, and other buildings aren't powered by little batteries: they're not supplied with DC current, but with alternating current (AC), which reverses its direction about 50 times per second (with a frequency of 50 Hz). If you want to run a motor from your household AC electricity supply, instead of from a DC battery, you need a different design of motor.

In an AC motor, there's a ring of electromagnets arranged around the outside (making up the **stator**), which are designed to produce a *rotating* magnetic field. Inside the stator, there's a solid metal axle, a loop of wire, a coil, a squirrel cage made of metal bars and interconnections (like the rotating cages people sometimes get to amuse pet mice), or some

other freely rotating metal part that can conduct electricity. Unlike in a DC motor, where you send power to the inner rotor, in an AC motor you send power to the outer coils that make up the *stator*. The coils are energized in pairs, in sequence, producing a magnetic field that rotates around the outside of the motor. The rotor, suspended inside the magnetic field, is an electrical conductor.

The magnetic field is constantly changing (because it's rotating) so, according to the laws of electromagnetism (Faraday's law, to be precise), the magnetic field produces (or induces, to use Faraday's own term) an electric current inside the rotor. If the conductor is a ring or a wire, the current flows around it in a loop. If the conductor is simply a solid piece of metal, eddy currents swirl around it instead. Either way, the induced current produces its own magnetic field and, according to another law of electromagnetism (Lenz's law) tries to stop whatever it is that causes it—the rotating magnetic field—by rotating as well. (You can think of the rotor frantically trying to "catch up" with the rotating magnetic field in an effort to eliminate the difference in motion between them.) Electromagnetic induction is the key to why a motor like this spins—and that's why it's called an induction motor. An electrical converts electrical energy into mechanical energy which is then supplied to different types of loads.

A.C. motors operates on A.C. supply, and they are classified into synchronous, single phase and three phase induction, and special purpose motors. Out of all types, three phase induction motors are most widely used for industrial applications mainly because they do not require a starting device. three phase induction motor derives its name from the fact that the rotor current is induced by the magnetic field, instead of electrical connection. The operation principle of a three phase induction motors is based on the production of rotating magnetic field .

16.Speed of RMF

The speed at which the rotating magnetic field revolves is called the synchronous speed (N_s). Referring to Fig. 8.9 the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, for P poles, the rotating field makes one revolution in $P/2$ cycles of current (see Fig. 8.7).

\therefore Cycles of current = $2/P \times$ revolutions of field

or Cycles of current per second = $2/P \times$ revolutions of field per second

Since revolutions per second is equal to the revolutions per minute (N_s) divided by 60

and the number of cycles per second is the frequency (f).

$$f = \frac{P}{2} * \frac{N_s}{60} = \frac{P N_s}{120}$$

$$N_s = \frac{120 f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

17. Slip

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the stator field speed (Ns). This difference in speed depends upon load on the motor. The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

$$\% \text{ age slip } S = \frac{N_s - N}{N_s} * 100$$

- (i) The quantity $N_s - N$ is sometimes called slip speed.
- (ii) When the rotor is stationary (i.e., $N = 0$), slip, $s = 1$ or 100 %.
- (iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

18. Rotor Frequency at operation condition

The frequency of a voltage or current induced due to the relative speed between a winding and a magnetic field is given by the general formula;

$$f = \frac{P N}{120}$$

Where

N = Relative speed between magnetic field and the winding

P = Number of poles

For a rotor speed N, the relative speed between the rotating flux and the rotor is $N_s - N$

Consequently, the rotor current frequency f_2 is given by;

$$f_2 = \frac{(N_s - N)P}{120}$$

$$f_2 = \frac{S N_s P}{120}$$

$$f_2 = S f_1$$

Where

f_2 = rotor current frequency,

S = slip and f_1 = supply frequency (stator frequency). The relative speed between the rotating field and stator winding is $N_s - 0 = N_s$.

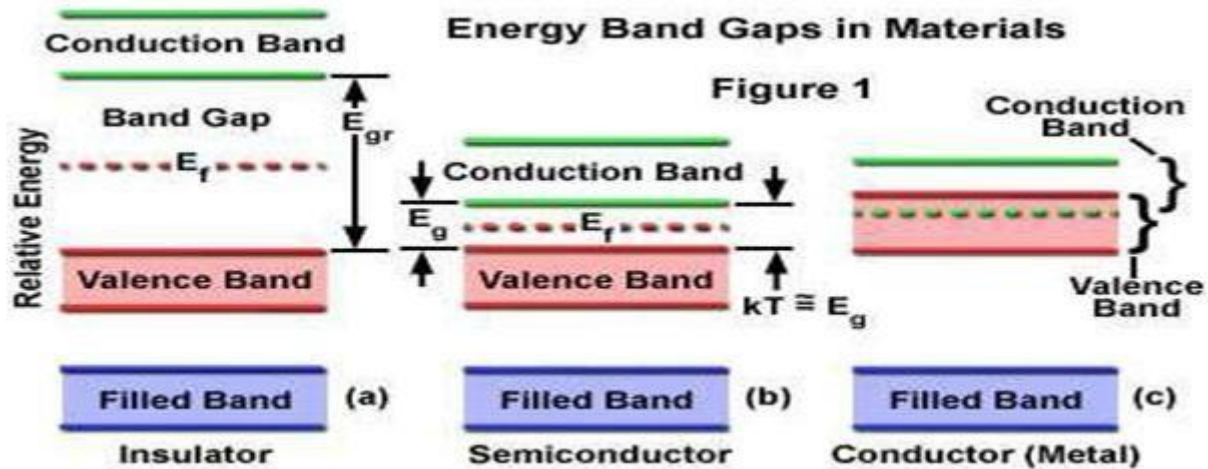
Therefore, the frequency of induced current or voltage in the stator winding is same as the supply frequency $f_1 = N_s P / 120$.

UNIT IV PN JUNCTION DIODE

4.1 INTRODUCTION:

We start our study of nonlinear circuit elements. These elements (diodes and transistors) are made of semiconductors. A brief description of how semiconductor devices work is first given to understand their $i-v$ characteristics. You will see a rigorous analysis of semiconductors in the breadth courses.

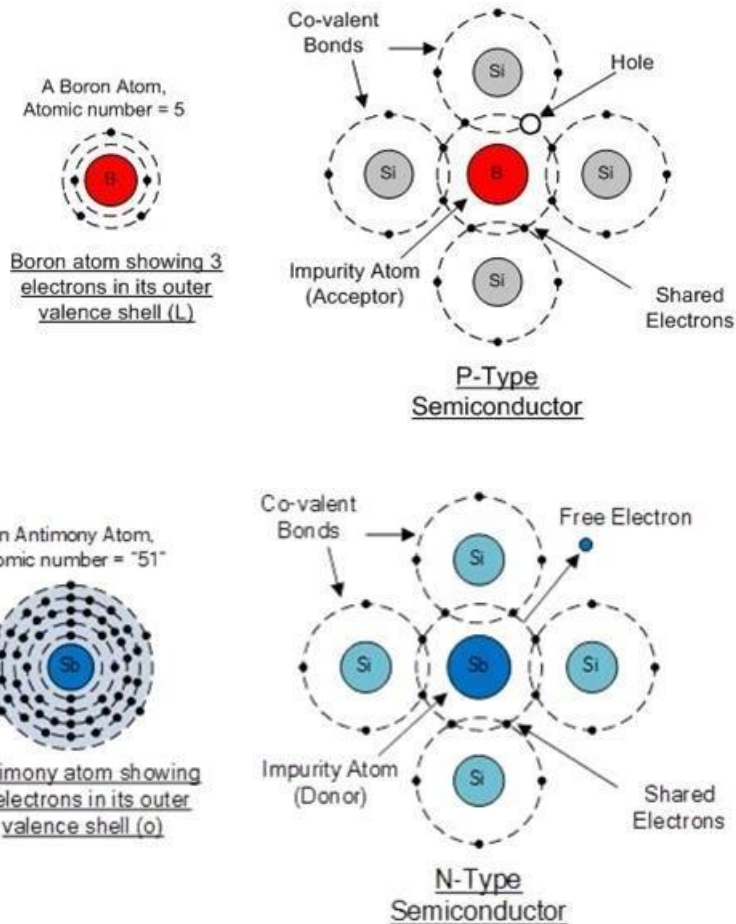
4.1.1 Energy Bands in Solids:



4.1.2 Semiconductors:

Semiconductor materials are mainly made of elements from group IVB of the periodic table like C (diamond), Si, Ge, SiC. These materials have 4 electrons in their outermost electronic shell. Each atom can form a "covalent" bond with four of its neighbors, sharing one electron with that atom. In this manner, each atom "sees" eight electrons in its outermost electronic shell (4 of its own, and one from each neighbor), completely filling that shell. It is also possible to form this type of covalent bond by combining elements from group IIIB (sharing three electrons) with an element from group VB (sharing five electrons). Examples of these semiconductors are GaAs or AlGaAs and are usually called "3-5" semiconductors. We focus mostly on Si semiconductors in this class. Figure below shows this covalent bond structure for Si. A pair of electrons and holes is also shown. Note that Si forms a tetrahedron structure and an atom in the center of the tetrahedron shares electrons with atoms on the each vertex. Figure below is a two-dimensional representation of such a structure. The left figure is for a pure Si semiconductor and an electron-hole pair is depicted. Both electrons and holes are called "mobile" carriers as they are responsible for carrying electric current.

If we add a small amount of an element from group VB, such as P, to the semiconductor, we create an n-type semiconductor and the impurity dopant is called an n-type dopant. Each of these new atoms also forms a covalent bond with four of its neighbors. However, as an n-type dopant has 5 valence electrons, the extra electron will be located in the "empty" energy band. As can be seen, there is no hole associated with this electron. In addition to electrons from the n-type dopant, there are electron-hole pairs in the solid from the base semiconductor (Si in the above figure) which are generated due to temperature effects. In an n-type semiconductor, the number of free electrons from the dopant is much larger than the number of electrons from electron-hole pairs. As such, an n-type semiconductor is considerably more conductive than the base semiconductor (in this respect, an n-type semiconductor is more like a "resistive" metal than a semiconductor).



In summary, in a n-type semiconductor there are two charge carriers: "holes" from the base semiconductor (called the "minority" carriers) and electrons from both the n-type dopant and electron-hole pairs (called the "majority" carrier).

Similarly, we can create a p-type semiconductor by adding an element from group IIIB, such as B, to the semiconductor. In this case, the p-type dopant generate holes. We will have two charge carriers: majority carriers are "holes" from the p-type dopant and electron-hole pairs and minority carriers are electrons from the base semiconductor (from electron-hole pairs).

The charge carriers (electrons and holes) move in a semiconductor through two mechanisms: First, charge carriers would move from regions of higher concentration to lower concentration in order to achieve a uniform distribution throughout the semiconductor. This process is called "Diffusion" and is characterized by the diffusion coefficient, D . Second, charge carriers move under the influence of an electricfield. This motion is called the drift and is characterized by the mobility.

4.2 DIODE WORKING PRINCIPLE

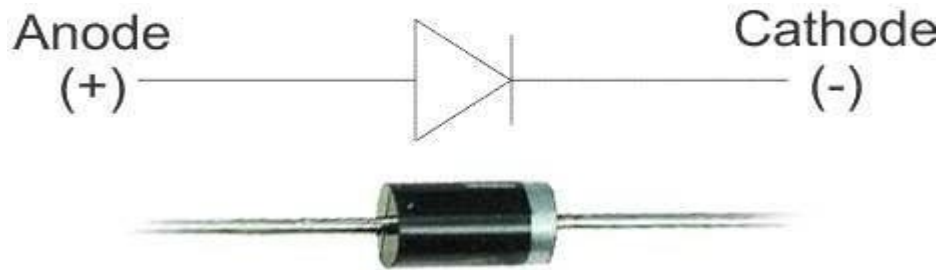
What is a Diode?

A diode is a device which only allows unidirectional flow of current if operated within a rated specified voltage level. A diode only blocks current in the reverse direction while the reverse voltage is within a limited range otherwise reverse barrier breaks and the voltage at which this breakdown occurs is called reverse breakdown voltage. The diode acts as a valve in the electronic and electrical circuit. A P-N junction is the simplest form of the diode which behaves as ideally short circuit when it is in forward biased and behaves as ideally open circuit when it is in the reverse biased. Beside simple PN junction diodes, there are different types of diodes although the fundamental principle is more or less same. So a particular arrangement of diodes can convert AC to pulsating DC, and hence, it is

sometimes also called as a rectifier. The name diode is derived from "di-ode" which means a device having two electrodes.

4.2.1 Symbol of Diode

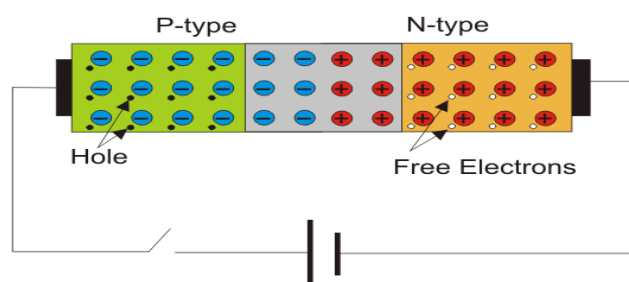
The symbol of a diode is shown below, the arrowhead points in the direction of conventional current flow.



A simple **PN junction diode** can be created by doping donor impurity in one portion and acceptor impurity in other portion of a silicon or germanium crystal block. These make a p n junction at the middle portion of the block beside which one portion is p type (which is doped by trivalent or acceptor impurity) and other portion is n type (which is doped by pentavalent or donor impurity). It can also be formed by joining a p-type (intrinsic semiconductor doped with a trivalent impurity) and n-type semiconductor (intrinsic semiconductor doped with a pentavalent impurity) together with a special fabrication technique such that a p-n junction is formed. Hence, it is a device with two elements, the p- type forms anode and the n-type forms the cathode. These terminals are brought out to make the external connections.

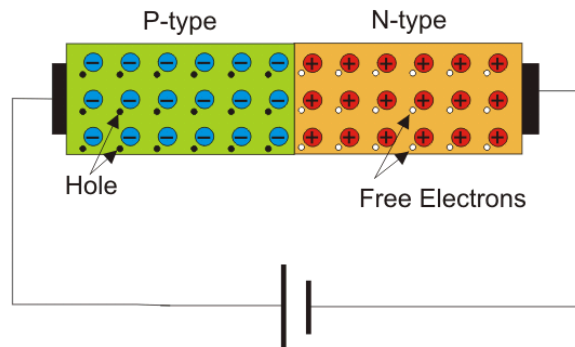
4.2.2 Working Principle of Diode

The n side will have a large number of electrons and very few holes (due to thermal excitation) whereas the p side will have a high concentration of holes and very few electrons. Due to this, a process called diffusion takes place. In this process free electrons from the n side will diffuse (spread) into the p side and combine with holes present there, leaving a positive immobile (not moveable) ion in the n side. Hence, few atoms on the p side are converted into negative ions. Similarly, few atoms on the n-side will get converted to positive ions. Due to this large number of positive ions and negative ions will accumulate on the n-side and p-side respectively. This region so formed is called as depletion region. Due to the presence of these positive and negative ions a static electric field called as "barrier potential" is created across the p-n junction of the diode. It is called as "barrier potential" because it acts as a barrier and opposes the further migration of holes and electrons across the junction.

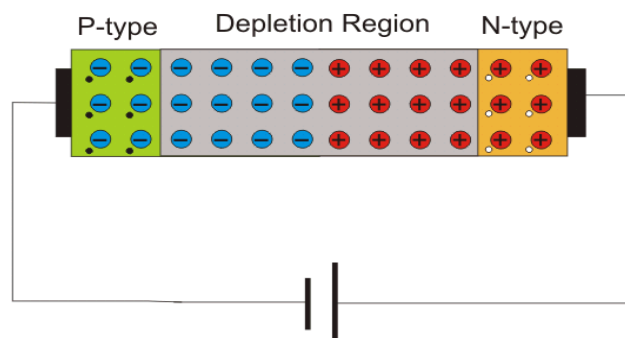


In a PN junction diode when the forward voltage is applied i.e. positive terminal of a source is connected to the p-type side, and the negative terminal of the source is connected to the n-type side, the diode is said to be in forward biased condition. We know that there is a barrier potential across the junction. This barrier potential is directed in the opposite of the forward applied voltage. So a diode can only allow current to flow in the forward direction when forward applied voltage is more than barrier potential of the junction. This voltage is called forward biased voltage. For silicon diode, it is 0.7 volts. For germanium diode, it is 0.3 volts. When forward applied voltage is more than this forward biased voltage, there will be forward current in the diode, and the diode will become short

circuited. Hence, there will be no more voltage drop across the diode beyond this forward biased voltage, and forward current is only limited by the external resistance" >resistance connected in series with the diode. Thus, if forward applied voltage increases from zero, the diode will start conducting only after this voltage reaches just above the barrier potential or forward biased voltage of the junction. The time taken by this input voltage to reach that value or in other words the time taken by this input voltage to overcome the forward biased voltage is called recovery time.

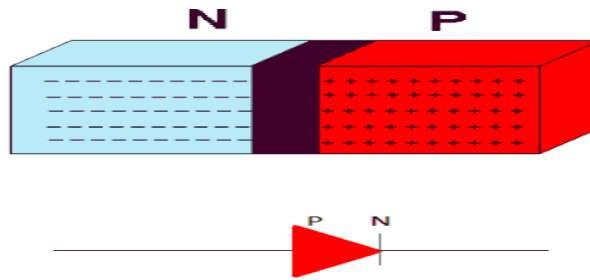


Now if the diode is reverse biased i.e. positive terminal of the source is connected to the n-type end, and the negative terminal of the source is connected to the p-type end of the diode, there will be no current through the diode except reverse saturation current. This is because at the reverse biased condition the depletion layer of the junction becomes wider with increasing reverse biased voltage. Although there is a tiny current flowing from n-type end to p-type end in the diode due to minority carriers. This tiny current is called reverse saturation current. Minority carriers are mainly thermally generated electrons and holes in p-type semiconductor and n-type semiconductor respectively. Now if reverse applied voltage across the diode is continually increased, then after certain applied voltage the depletion layer will destroy which will cause a huge reverse current to flow through the diode. If this current is not externally limited and it reaches beyond the safe value, the **diode** may be permanently destroyed. This is because, as the magnitude of the reverse voltage increases, the kinetic energy of the minority charge carriers also increase. These fast moving electrons collide with the other atoms in the device to knock-off some more electrons from them. The electrons so released further release much more electrons from the atoms by breaking the covalent bonds. This process is termed as carrier multiplication and leads to a considerable increase in the flow of current through the p-n junction. The associated phenomenon is called Avalanche Breakdown.



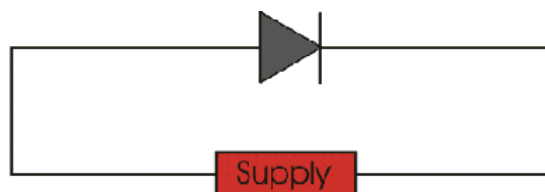
4.3 DIODE CHARACTERISTICS

Semiconductor materials (Si, Ge) are used to form variety of electronic devices. The most basic device is diode. Diode is a two terminal P-N junction device. P-N junction is formed by bringing a P type material in contact with N type material. When a P-type material is brought in contact with N- type material electrons and holes start recombining near the junction. This result in lack of charge carriers at the junction and thus the junction is called depletion region. Symbol of P-N junction is given as:



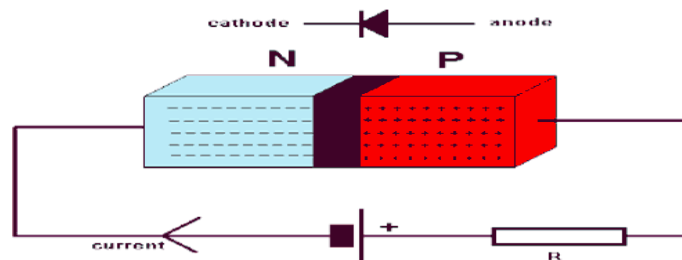
Biased i.e. when voltage is applied across the terminals of P-N junction, it is called diode.

Diode is unidirectional device that allows the flow of current in one direction only depending on the biasing.



4.3.1 Forward Biasing Characteristic of Diode

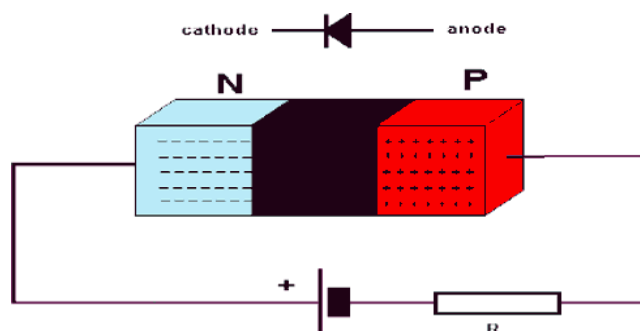
When, P terminal is more positive as compared to N terminal i.e. P- terminal connected to positive terminal of battery and N-terminal connected to negative terminal of battery, it is said to be forward biased.



Positive terminal of the battery repels majority carriers, holes, in P-region and negative terminal repels electrons in the N-region and push them towards the junction. This result in increase in concentration of charge carriers near junction, recombination takes place and width of depletion region decreases. As forward bias voltage is raised depletion region continues to reduce in width, and more and more carriers recombine. This results in exponential rise of current.

4.3.2 Reverse Biasing Characteristic of Diode

In reverse biasing P- terminal is connected to negative terminal of the battery and N- terminal to positive terminal of battery. Thus applied voltage makes N-side more positive than P-side.



Negative terminal of the battery attracts majority carriers, holes, in P-region and positive terminal attracts electrons in the N-region and pull them away from the junction. This result in decrease in concentration of charge carriers near junction and width of depletion region increases. A small amount of current flow due to minority carriers, called as reverse bias current or leakage current. As reverse bias voltage is raised depletion region continues to increase in width and no current flows. It can be concluded that diode acts only when forward biased. Operation of diode can be summarized in form of I-V **diode characteristics** graph. For reverse bias diode, $V < 0$, $I_D = I_S$ Where, V = supply voltage I_D = diode current I_S = reverse saturation current For forward bias, $V > 0$, $I_D = I_S(e^{V/NV_T} - 1)$

Where,

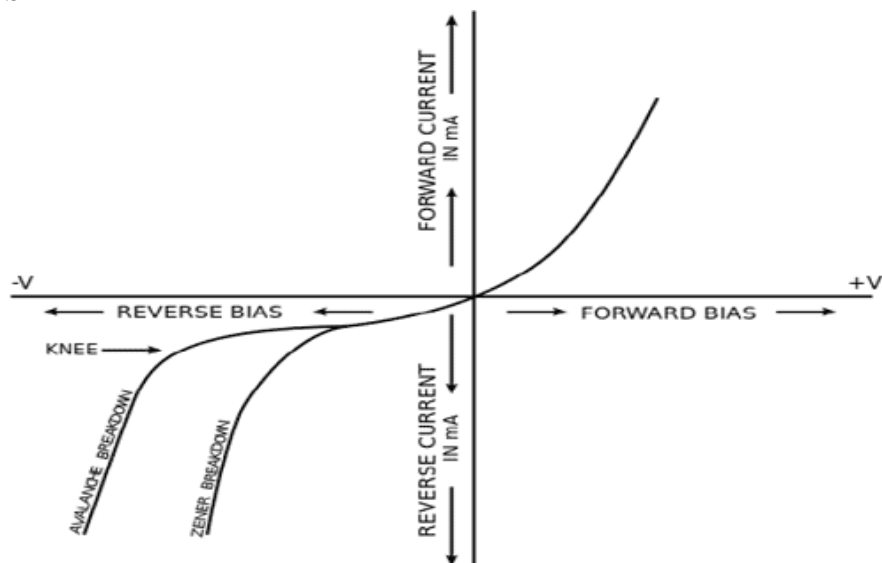
V_T = volt's equivalent of temperature = $KT/Q = T/11600$

Q = electronic charge = $1.632 \times 10^{-19} \text{ C}$

K = Boltzmann's constant = 1.38×10^{-23}

$N = 1$, for Ge

= 2, for Si



As reverse bias voltage is further raised, depletion region width increases and a point comes when junction breaks down. This results in large flow of current. Breakdown is the knee of **diode characteristics** curve. Junction breakdown takes place due to two phenomena

4.3.3 Avalanche Breakdown(for $V > 5V$)

Under very high reverse bias voltage kinetic energy of minority carriers become so large that they knock out electrons from covalent bonds, which in turn knock more electrons and this cycle continues until and unless junction breakdowns.

4.3.4 Zener Effect (for $V < 5V$)

Under reverse bias voltage junction barrier tends to increase with increase in bias voltage. This results in very high static electric field at the junction. This static electric field breaks covalent bond and set minority carriers free which contributes to reverse current. Current increases abruptly and junction breaks down.

4.4 P-N JUNCTION DIODE AND CHARACTERISTICS OF P-N JUNCTION

The volt-ampere characteristics of a diode explained by the following equations:

$$I = I_S(e^{V_D/(\eta V_T)} - 1)$$

Where

I = current flowing in the diode,

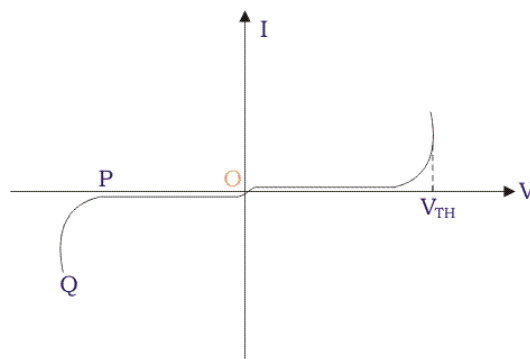
I_0 = reverse saturation current

V_D = Voltage applied to the diode

V_T = volt- equivalent of temperature = $k T/q = T/ 11,600 = 26\text{mV}$ (@ room temp)

$\eta = 1$ (for Ge) and 2 (for Si)

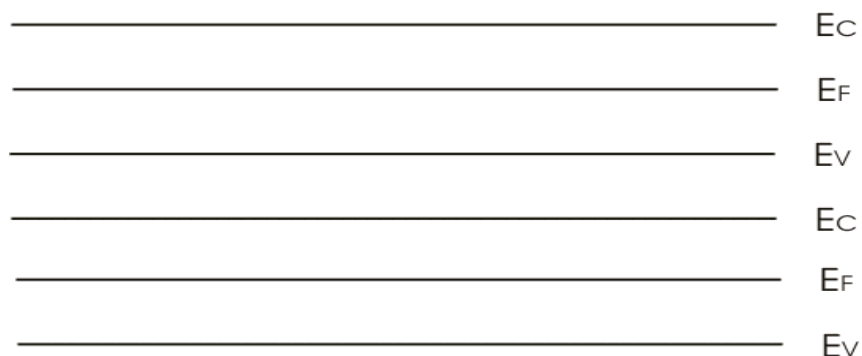
It is observed that **Ge** diodes has smaller cut-in-voltage when compared to **Si** diode. The reverse saturation current in **Ge** diode is larger in magnitude when compared to silicon diode.



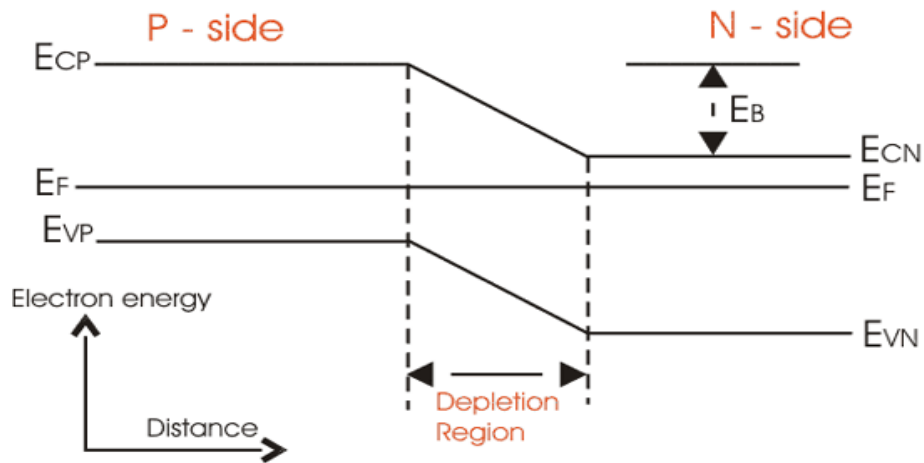
When, V is positive the junction is forward biased and when V is negative, the junction is reversing biased. When V is negative and less than V_{TH} , the current is very small. But when V exceeds V_{TH} , the current suddenly becomes very high. The voltage V_{TH} is known as threshold or cut in voltage. For Silicon diode $V_{TH} = 0.6 \text{ V}$. At a reverse voltage corresponding to the point P , there is abrupt increment in reverse current. The PQ portion of the characteristics is known as breakdown region.

4.5 P-N Junction Band Diagram

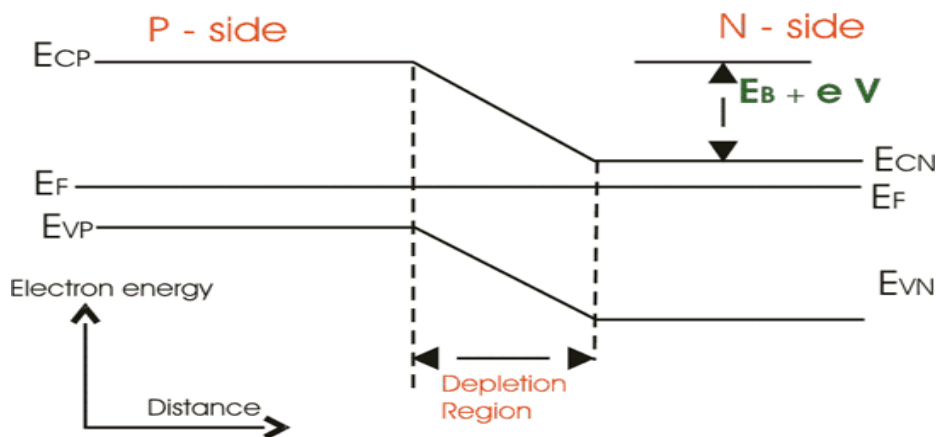
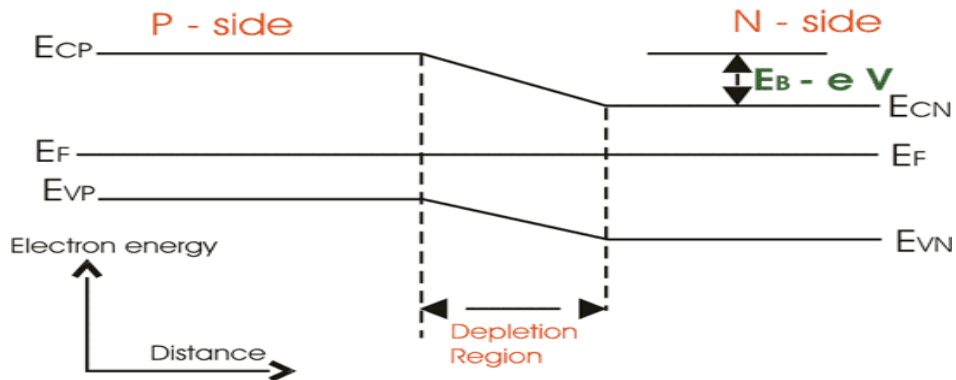
For an n-type semiconductor, the Fermi level E_F lies near the conduction band edge. E_C but for an p - type semiconductor, E_F lies near the valance band edge E_V .



Now, when a p-n junction is built, the Fermi energy E_F attains a constant value. In this scenario the p- sides conduction band edge. Similarly n-side valance band edge will be at higher level than E_{cn} , n- sides conduction band edge of p - side. This energy difference is known as barrier energy. The barrier energy is $E_B = E_{cp} - E_{cn} = E_{vp} - E_{vn}$



If we apply forward bias voltage V , across junction then the barrier energy decreases by an amount of eV and if V is reverse bias is applied the barrier energy increases by eV .



2.4. Diode Current Equation

The mathematical equation, which describes the forward and reverse characteristics of a semiconductor diode is called the **diode current equation**.

Let I = Forward (or reverse) diode current,

I_{RS} = Reverse saturation current,

V = External voltage (It is positive for forward bias and negative for reverse bias),

η = A constant

= 1 for germanium diodes, 2 for silicon diodes for relative low value of diode current (i.e., at or below the knee of the curve)

= 1 for germanium and silicon for higher levels of diode current (i.e., in the rapidly increasing section of the curve), and

V_T = Volt-equivalent of temperature. Its value is given by the relation, $\frac{T}{11600}$,

where T is the absolute temperature

= 26 mV at room temperature (300 K).

For a forward-biased diode, the current equation is given by the relation,

$$I = I_{RS} [e^{V/(\eta \times V_T)} - 1] \quad \dots(i)$$

Substituting the value of $V_T = 26$ mV or 0.026 V (at room temperature) in eqn. (i), we get

$$I = I_{RS} (e^{40V/\eta})$$

\therefore Diode current at or below the knee, for germanium,

$$I = I_{RS} (e^{40V} - 1) \quad (\because \eta = 1)$$

and, for silicon,

$$I = I_{RS} (e^{20V} - 1) \quad (\because \eta = 2)$$

When the value of applied voltage is greater than unity (i.e., for the diode current in the rapidly increasing section of curve), the equation of diode current for germanium or and silicon,

$$I = I_{RS} \cdot e^{20V} \quad (\because \eta = 2)$$

The current equation for a reverse biased diode may be obtained from eqn. (i) by changing the sign of the applied voltage (V). Thus the diode current for reverse bias,

$$I = I_{RS} [e^{-V/(\eta \times V_T)} - 1]$$

When $V \gg V_T$, then the term $e^{-V/(\eta \times V_T)} \ll 1$. Therefore $I = I_{RS}$. Thus the diode current under reverse bias is equal to the reverse saturation current as long as the external voltage is below its breakdown value.

4.5.1 APPLICATIONS OF DIODES

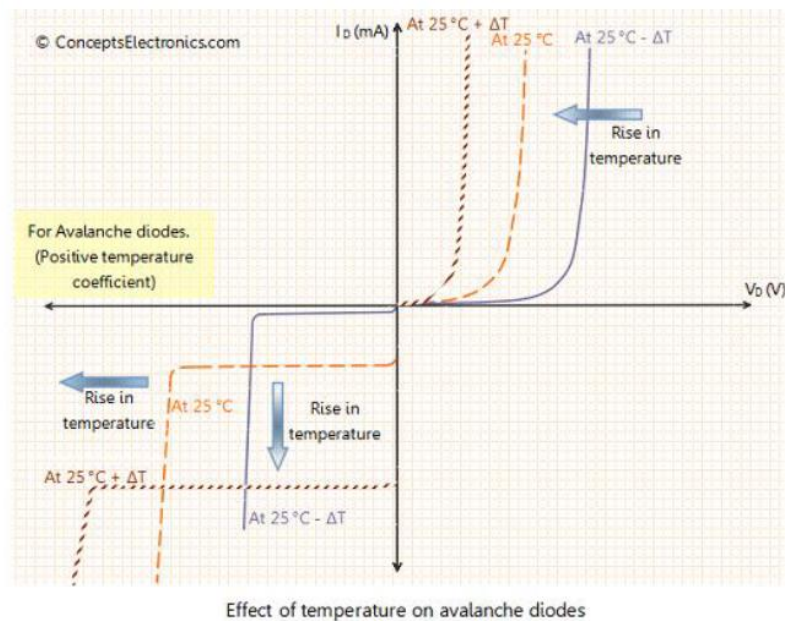
- Rectifying a voltage, such as turning AC into DC voltages
- Isolating signals from a supply
- Voltage Reference
- Controlling the size of a signal
- Mixing signals
- Detection signals
- Lighting
- Lasers diodes

Effect of temperature on PN junction diode.

- PN junction diode parameters like cut in voltage , forward current in the forward bias and reverse current, reverse breakdown voltage and reverse saturation current in the reverse bias are dependent on temperature.
- The current that a PN junction diode can conduct at a given voltage is dependent upon the operating temperature. An increased temperature will result in a large number of broken covalent bonds increasing the large number of majority and minority carriers. Rise in temperature generates more electron-hole pair thus conductivity increases and thus increase in current. This amounts to a diode current larger than its previous diode current. The above phenomenon applies both to forward and reverse current.
- Mathematically diode current is given by

$$I = I_{RS} [e^{V/(\eta \times V_T)} - 1]$$

Hence from equation we conclude that the current should decrease with increase in temperature but exactly opposite occurs, there are two reasons:



Forward bias region :

- The effect of increased temperature and decrease in temperature on the forward characteristics curve of a PN junction diode is as shown in above figure. It may be noted that the forward characteristics of silicon diode shift to the left at rate of 2.5 m V per centigrade degree increase in temperature and shift towards right at rate of 2.5 m V per centigrade degree decrease in temperature.
- The cut-in voltage decreases as the temperature increases. The diode conducts at smaller voltage at large temperature.
- The cut-in voltage increases as the temperature decreases. The diode conducts at larger voltage at lower temperature.

Example:

At 25° C $V_D = 0.7V$

for 100° C Rise in temp. i.e., at 25+100=125° C

now we will find new V_D at 125° C then $100 \times 2.5mV = 0.25V$

the new V_D will be reduce by 0.25 V

therefore new $V_D = 0.7 - 0.25 = 0.45V$

At 25° C $V_D = 0.7V$

for 100° C decrease in temp. i.e., at 25-100= -75° C

now we will find new V_D at 75° C then $100 \times 2.5mV = 0.25V$

the new V_D will be increase by 0.25 V

therefore new $V_D = 0.7 + 0.25 = 0.95V$

Reverse bias region:

- The effect of increased temperature and decrease in temperature on the reverse characteristics curve of a PN junction diode is as shown in above figure. It may be noted that in the reverse bias region characteristics reverse current of silicon diode shift downwards with the increase in temperature and shift upward with decrease in temperature.
- In the reverse bias region the reverse current of diode doubles for every 10°C rise in temperature.

. Example:

- For 100° C Rise in temp. i.e., at 25+100=125°C

25° C → 10nA

35° C → 20nA

45° C → 40nA

55° C → 80nA

.

.

.

.

.

.

125° → 10240n A (OR) 10.24 μ A

Therefore for 100° C Rise in temp. i.e., at 25+100=125°C the reverse saturation current increases to greater than 10nA

- For 100° C decrease in temp. i.e., at 25-100 = -75°C the reverse saturation current reduces to less than 10nA

Static and Dynamic Resistance of a Diode

Static forward resistances (R_F). A diode has a definite value of resistance when forward biased. It is given by the *ratio of the D.C. voltage across the diode to D.C. current flowing through it.*

$$\text{Mathematically, } R_F = \frac{V_F}{I_F}$$

R_F may be obtained *graphically* from the diode forward characteristics as shown in Fig. . From the operating point P, the static forward resistance,

$$R_F = \frac{0.8}{16} = 0.05 \Omega.$$

Dynamic or A.C. resistance. In practice we don't use static forward resistance, instead, we use the dynamic or A.C. resistance. The A.C. resistance of a diode, at a particular D.C. voltage, is *equal to the reciprocal of the slope of the characteristic at that point*; *i.e.*, the A.C. resistance,

$$r_{AC} = \frac{1}{\Delta I_F / \Delta V_F} = \frac{\Delta V_F}{\Delta I_F} = \frac{\text{Change in voltage}}{\text{Resulting change in current}}.$$

Owing to the non-linear shape of the forward characteristic, the value of A.C. resistance of a diode is in the range of 1 to 25 Ω . Usually it is *smaller than D.C. resistance of a diode.*

Reverse resistance. When a diode is *reverse biased*, besides the forward resistance, it also possesses another resistance known as *reverse resistance*. It can be either D.C. or A.C. depending upon whether the reverse bias is direct or alternating voltage. Ideally, the reverse resistance of a diode is infinite. However, in actual practice, the reverse resistance is never infinite. It is *due to the existence of leakage current* in a reverse biased diode.

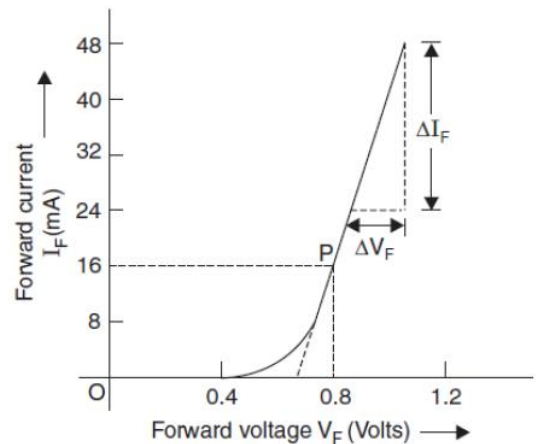
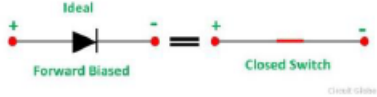
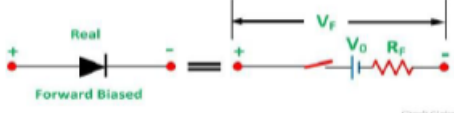
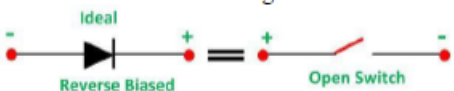

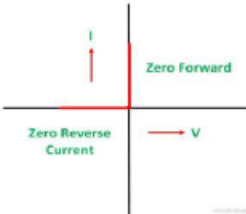
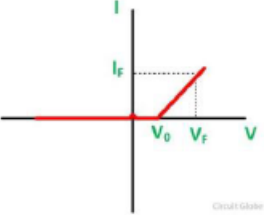


Fig. Static and dynamic forward resistances of a diode from the characteristic curve.

Ideal Diode and Practical Diode

Ideal Diode	Practical Diode
<p>1) A diode is said to be an Ideal Diode when it is forward biased and acts like a perfect conductor, with zero voltage across it. Similarly, when the diode is reversed biased, it acts as a perfect insulator with zero current through it.</p>	<p>1) A Practical diode contains barrier potential V_0 (0.7 V for silicon and 0.3 V for Germanium) and a forward resistance R_F of about 25 ohms. When a diode is forward biased and conducts a forward current I_F flows through it which causes a voltage drop $I_F R_F$ in the forward resistance. Hence, the forward voltage V_F applied across the Practical diode for conduction, has to overcome the following.</p> <ul style="list-style-type: none"> (i) Potential barrier (ii) Drop in forward resistance <p>i.e.,</p> $V_F = V_0 + I_F R_F$
<p>2) When the Ideal diode is forward biased it acts like a closed switch as shown in the figure below. An Ideal diode also acts like a switch</p> <div style="text-align: center;">  </div> <p>Diode resistance is zero i.e., $R_D=0$ from ohms law $V_D = I_D R_D$ Therefore $V_D=0$</p>	<p>2) The equivalent circuit of the Practical diode under forwarding bias condition is shown below. This circuit shows that a Practical diode still acts as a switch when forward biased, but the voltage required to operate this switch is V_F</p> <div style="text-align: center;">  </div> $V_F = V_0 + I_F R_F$
<p>3) Where as, if the diode is reversed biased, it acts like an open switch as shown in the figure below.</p> <div style="text-align: center;">  </div> <p>Diode resistance is infinity i.e., $R_D=\infty$ from ohms law $V_D = I_D R_D$ Therefore $I_D=0$</p>	<p>3) For all the practical purposes, a diode is considered to be an open switch when reversing biased. It is because the value of reverse resistance is so high ($R_R > 100 \text{ M}\Omega$) that is considered to be an infinite for all practical purposes.</p> <div style="text-align: center;">  </div>
<p>4) The V-I characteristics of the Ideal diode are shown in the figure below</p> <div style="text-align: center;">  </div>	<p>4) The V-I characteristic of the Practical diode is shown below.</p> <div style="text-align: center;">  </div>

Equivalent Diode Circuits

An equivalent circuit is nothing but a combination of elements that best represents the actual terminal characteristics of the device. In simple language, it simply means the diode in the circuit can be replaced by other elements without severely affecting the behavior of circuit.

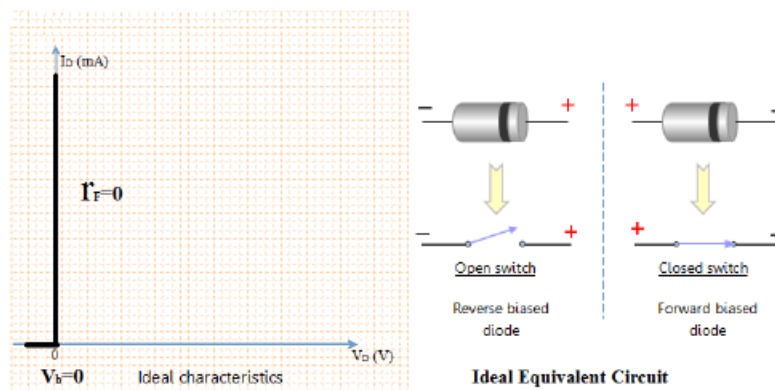
The diode can be modeled in three different ways depending on the accuracy required. Three models with increasing accuracy are listed below:

1. Ideal Diode Equivalent Circuit:
2. Constant voltage drop (or) Simplified Equivalent Circuit
3. Piece-Wise Linear Equivalent Circuit

1. Ideal Diode Equivalent Circuit:

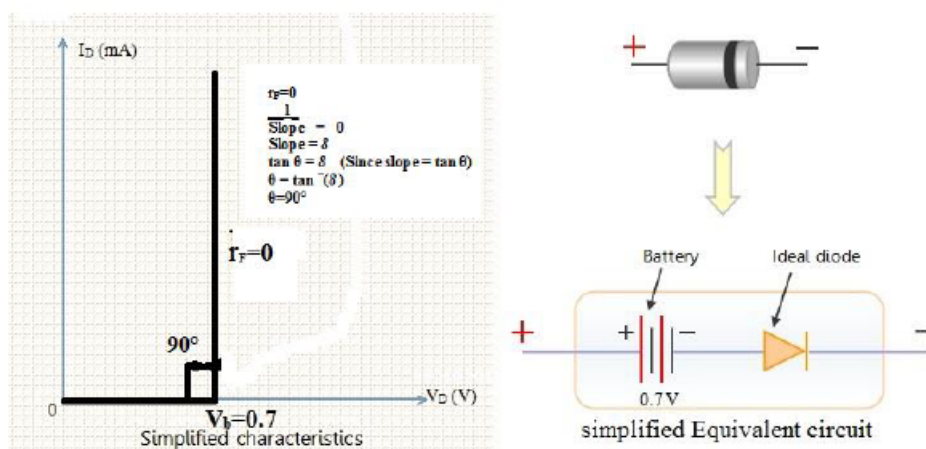
Figure indicates that the voltage drop across the diode is zero for any value of diode current. The ideal diode does not allow any current to flow in reverse biased condition. The current flowing through the diode is zero for any value of reverse biased voltage. Taking this into consideration, the ideal diode can be modeled as open or closed switch depending on the bias voltage.

- a) Ideal diode allows the flow of forward current for any value of forward bias voltage. Hence, Ideal diode can be modeled as closed switch under forward bias condition. This is shown in the figure.
- b) Ideal diode allows zero current to flow under reverse biased condition. Hence it can be modeled as open switch. This is indicated in the figure.



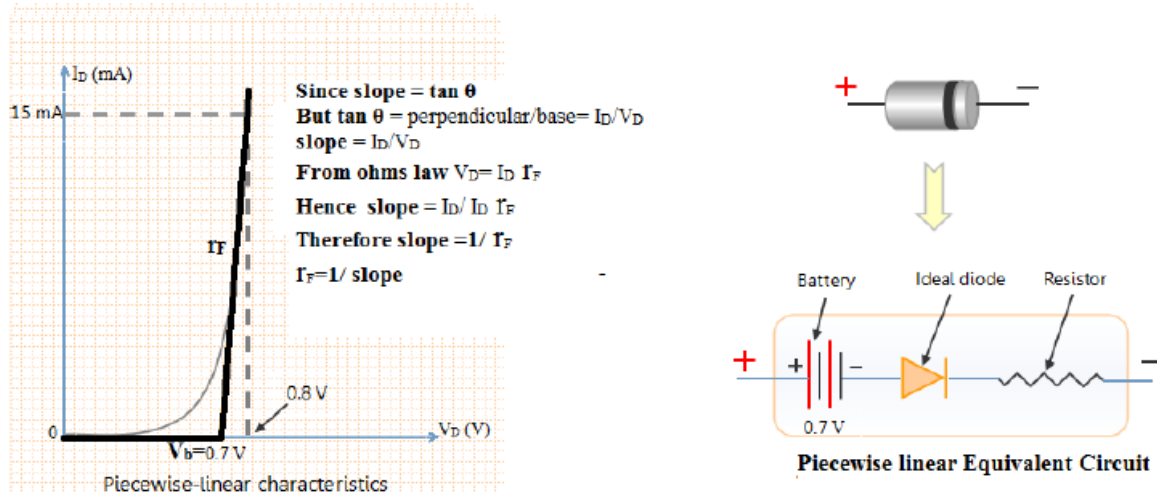
2. Constant Voltage Drop (or) Simplified Equivalent Circuit

The equivalent circuit in this case consists of a battery and an ideal diode. Consider the horizontal line from (0 to 0.7 V) in the curve. The horizontal line indicates that the current flowing through diode is zero for voltages between 0 and 0.7 V. To model this behavior, we put a battery of 0.7 V in the equivalent diode model. This does not mean that diodes are a source of voltage. When you measure the voltage across an isolated diode, the instrument will show zero value. The battery simply indicates that it opposes the flow of current in forward direction until 0.7 V. As the voltage becomes larger than 0.7 V, the current starts flowing in forward direction.



3. Piece-Wise Linear Equivalent Circuit

The piece-wise linear circuit, as the name suggests, is a model in which the characteristics of diode is approximated by "piece-wise linear" line segments. Now consider the straight line in the piece-wise linear characteristics. This straight line indicates constant slope. Slope in the V-I graph indicates resistance. So we add a resistor in the diode model. The value of resistance can be found from the graph. We can see from the graph that the diode current changes from 0 to 15 mA for a voltage change from 0.7 to 0.8 V. Thus the average value of resistance is $(0.8 \text{ V} - 0.7 \text{ V}) / (15 \text{ mA} - 0 \text{ mA}) = 6.67 \Omega$. Thus the value of resistance in the equivalent model is approximately 6.67Ω . The figure given below shows piece-wise linear characteristics of diode along with its model.



In the graph shown on left, the actual characteristics of diode is superimposed by piece-wise linear characteristics (shown in amber color). It is clear that the piece-wise linear characteristics do not exactly represent the characteristics of diode, especially near the knee of the curve. However it provides a good first approximation to the actual characteristics of the diode. Piece- wise linear characteristics can be obtained by replacing the diode in the circuit with a resistor, a battery and an ideal diode. This is shown in the right side of the above figure.

ZENER DIODE

A properly doped P-N junction crystal diode which has a sharp breakdown voltage is known as **Zener diode**.

The voltage-regulator diode is commonly called a '**Zener**' diode. It is a voltage limiting diode that has some applications in common with the older voltage-regulator gas tubes but serves a much wider field of application, because the devices cover a wide spectrum of voltages and power levels.

Performance/Operation

The electrical performance of a zener diode is based on the *avalanche characteristics* of the P-N junction. When a source of voltage is applied to a diode in the *reverse direction* (negative to anode), a reverse current I_R is observed (see Fig.). As the reverse potential is increased beyond the "Zener knee" avalanche breakdown becomes well developed at zener voltage V_Z . At voltage V_Z , the high counter resistance drops to a low value and the junction current increases rapidly. The current must of necessity be limited by an external resistance, since the voltage V_Z developed across the zener diode remains essentially constant. *Avalanche breakdown of the operating zener diode is not destructive as long as the rated power dissipation of the junction is not exceeded.*

Externally, the zener diode looks much like other silicon rectifying devices, and electrically it is capable of rectifying alternating current.

The following points about the *Zener diode* are worth noting :

(i) It looks like an ordinary diode except that it is properly doped so as to have a sharp breakdown voltage.

(ii) It is always reverse connected *i.e.*, it is *always reverse biased*.

(iii) It has sharp breakdown voltage, called Zener voltage V_Z .

(iv) When forward biased, its characteristics are just those of ordinary diode.

(v) It is not immediately burnt just because it has entered the breakdown region (The current is limited only by both external resistance and power dissipation of Zener diode).

- The location of Zener region can be controlled by varying the doping levels. An increase in doping, producing an increase in the number of added impurities, will decrease the Zener potential.

- Zener diodes are available having Zener potentials of 1.8 to 200 V with power ratings from $\frac{1}{4}$ to 50 W. Because of its higher temperature and current capability, silicon is usually preferred in the manufacture of Zener diodes.

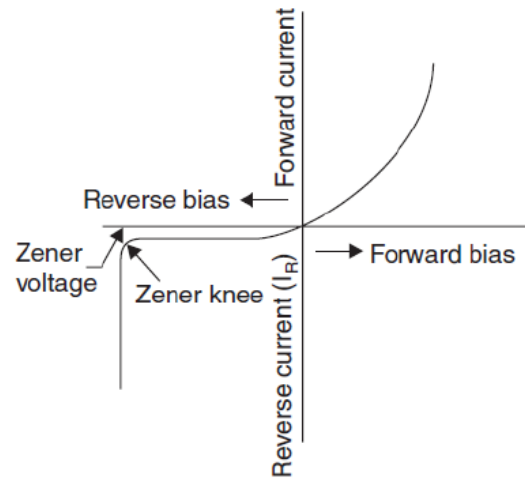


Fig.

Equivalent Circuit of Zener Diode

The complete equivalent circuit of the Zener diode in the Zener region includes a small dynamic resistance and D.C. battery equal to the Zener potential, as shown in Fig. :

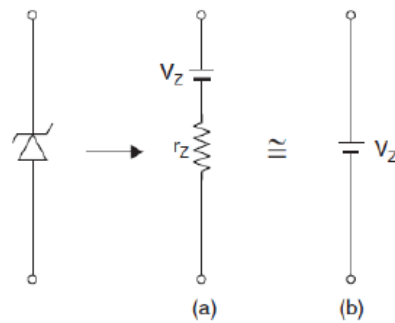


Fig. Zener equivalent circuit : (a) Complete ; (b) Approximately.

“ON” state. When reverse voltage across a Zener diode is equal to or more than breakdown voltage V_Z , the current increases very sharply. In this region curve is almost vertical ; it means that voltage across Zener diode is constant at V_Z even though the current through it changes. Therefore, in the breakdown region, an ideal Zener diode (this assumption is fairly reasonable as the impedance of Zener diode is quite small in the breakdown region) can be represented by a battery of voltage V_Z as shown in Fig. (b). Under such conditions, the Zener diode is said to be in the “ON” state.

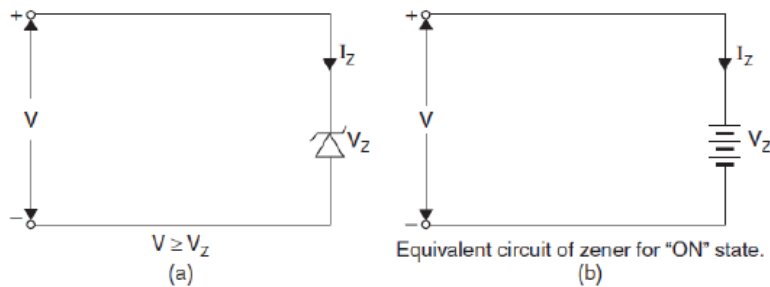
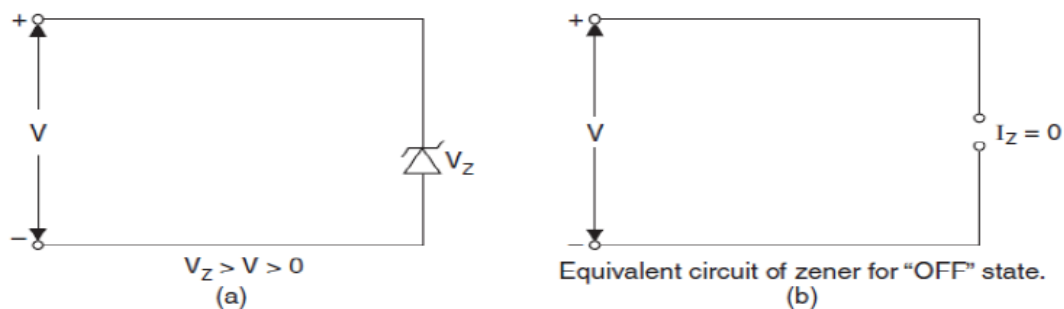


Fig. :

“OFF” state. When the reverse voltage across the Zener diode is less than V_Z but greater than 0 V, the Zener diode is in the “OFF” stage. Under such conditions, the Zener diode can be represented by an open circuit as shown in Fig. (b).



3.3. Applications of Zener Diode

Zener diode serves in the following variety of applications :

1. Voltage reference or regulator element :

The primary use of a zener diode is as a *voltage reference or regulator element*. Fig. 22 shows the fundamental circuit for the Zener diode employed as a shunt regulator. In the circuit, diode element and load R_L draw current through the series resistance R_S . If E_{in} increases, the current through the Zener element will increase and thus maintain an essentially fixed voltage across R_L . This ability to maintain the desired voltage is determined by the temperature coefficient and the diode impedance of the zener device.

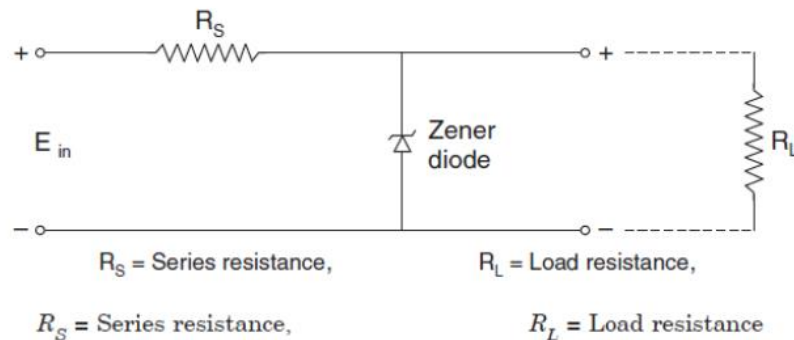


Fig. 22. Basic Zener-diode regulator circuit.

2. Shunt transistor regulator :

The Zener diode may also be used to control the reference voltage of a transistor regulated power supply. An example of this in a shunt transistor regulator is shown in Fig. 23, where Zener element is used to control the operating point of the transistor. The advantage of this circuit over that shown in Fig. 22 are *increased power handling capability and a regulating factor improved by utilizing the current gain of the transistor*.

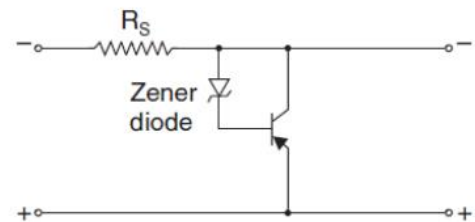


Fig. 23. Shunt transistor regulator.

3. Audio or r-f applications :

The Zener diode also finds use in audio or *r-f* (radio frequency) applications where a source of stable reference voltage is required, as in bias supplies. Frequently, *Zener diodes are connected in series package*, with, for example, one junction operating in the reverse within a single direction and possessing a positive temperature V_Z coefficient ; the remaining diodes are connected to operate in the forward direction and exhibit negative temperature V_Z coefficient characteristics. The net result

3.22. Zener Diode Regulator

The major application of zener diode in the electronic circuit is as a voltage regulator. It provides a constant voltage to the load from a source whose voltage may vary over sufficient range. The zener diode of zener voltage V_z is reverse connected across the load R_L across which constant voltage is desired. A resistor R is connected in series with the circuit which absorbs the output voltage fluctuation so as to maintain constant voltage (V_0) across the load.

Let a variable voltage V_{in} be applied across the load R_L . When the value of V_{in} is less than zener voltage V_z of the zener diode. No current flows through it and the same voltage appears across the load. When the input voltage V_{in} is more than V_z this will cause the zener diode to conduct a large current I_z .

In the above discussion it has been seen that when a zener diode of zener voltage V_z is connected in reverse direction parallel to the load. It maintains a constant voltage across the load equal to V_z and hence stabilises the output voltage.

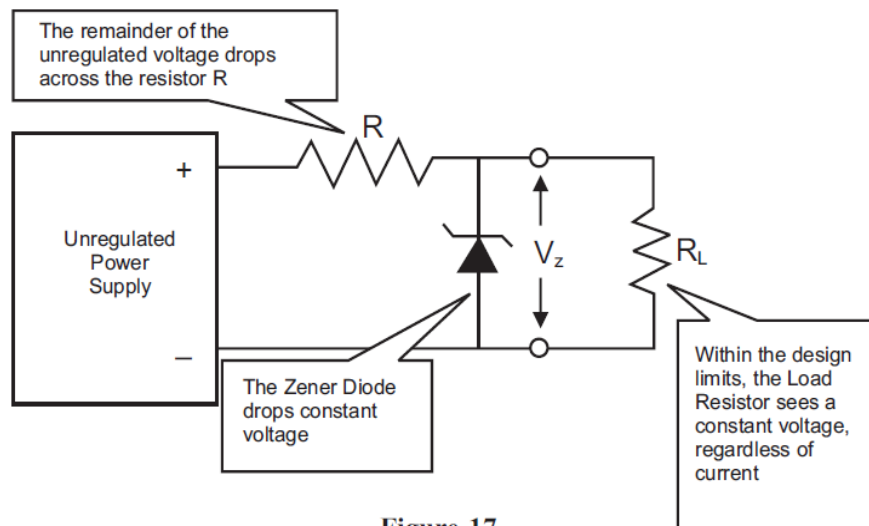


Figure-17

Example 4. Determine the current flowing through the Zener diode for the circuit shown in Fig. 24, if $R_L = 4000 \Omega$, input voltage is 50 volts, $R_S = 1800 \Omega$ and output voltage is 32 volts.

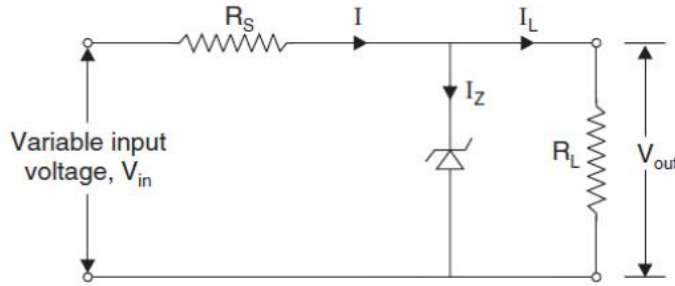


Fig. 24

Solution. Input voltage, $V_{in} = 50 \text{ V}$
 Output voltage, $V_{out} = 32 \text{ V}$
 Voltage drop in series resistor, $R_S = V_{in} - V_{out} = 50 - 32 = 18 \text{ V}$

Current through series resistance, $I = \frac{V_{in} - V_{out}}{R} = \frac{18}{1800} = .01 \text{ A or } 10 \text{ mA}$

Load current, $I_L = \frac{V_{out}}{R_L} = \frac{32}{4000} = 0.008 \text{ A or } 8 \text{ mA}$

Current through Zener diode, $I_Z = I - I_L = 10 - 8 = 2 \text{ mA. (Ans.)}$

Example 5. Determine the maximum and minimum values of Zener current if in the circuit shown in Fig. 24 the load resistance, $R_L = 4000 \Omega$, series resistance = 8000Ω , output voltage = 32 V and source voltage varies between 100 V and 128 V.

Solution. Refer to Fig. 23. Given : $R_L = 4000 \Omega$; $R = 8000 \Omega$; $V_{out} = 32 \text{ V}$;

Load current, $I_L = \frac{V_{out}}{R_L} = \frac{32}{4000} = 0.008 \text{ A or } 8 \text{ mA}$

The Zener current will be maximum when input voltage is maximum i.e., 128 V

Corresponding current through series resistance,

$$I = \frac{V_{in(max)} - V_{out}}{R_S} = \frac{128 - 32}{8000} = 0.012 \text{ A or } 12 \text{ mA}$$

Corresponding Zener current, $(I_Z)_{max} = I - I_L = 12 - 8 = 4 \text{ mA. (Ans.)}$

The Zener current will be minimum when input voltage is minimum i.e., 100 V.

Corresponding, current through series resistance,

$$I' = \frac{V_{in(min)} - V_{out}}{R_S} = \frac{100 - 32}{8000} = 0.0085 = 8.5 \text{ mA}$$

Corresponding Zener current, $(I_Z)_{min} = I' - I_L = 8.5 - 8 = 0.5 \text{ mA. (Ans.)}$

Example 6. In the simple Zener-diode based voltage regulator shown in Fig. 25, a 5.6 V, 0.25 W Zener diode is used. For reliable operation, the minimum I_Z should be 1 mA. The load R_L varies between 20Ω and 50Ω . Find the range of R_S for reliable and safe operation of the voltage regulator.

Solution. (i) Let $R_S = 20 \Omega$

$$I = \frac{5.6}{20} = 0.28 \text{ A}$$

$$R_S = \frac{10 - 5.6}{0.28 + 0.001} = 15.66 \Omega \approx 16 \Omega.$$

(ii) Let,

$$R_S = 50 \Omega$$

$$I = \frac{5.6}{50} = 0.112 \text{ A}$$

$$R_S = \frac{10 - 5.6}{0.112 + 0.001} = 38.93 \Omega \approx 39 \Omega.$$

$\therefore R$ ranges from 16Ω to 39Ω . (Ans.)

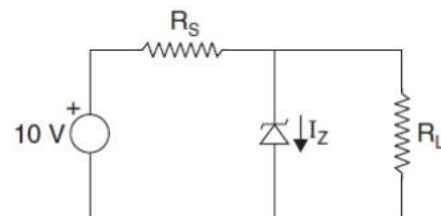


Fig. 25

Diode Junction capacitance:

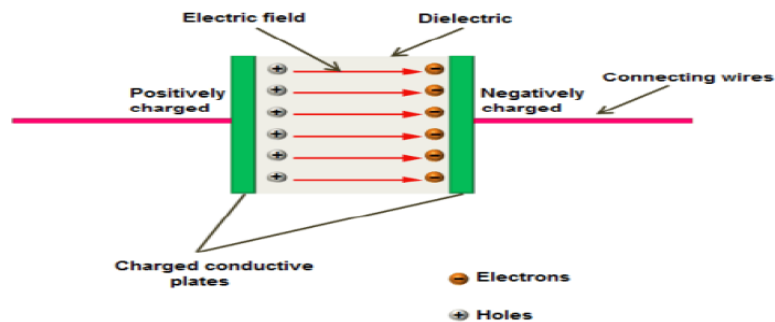
In a p-n junction diode, two types of capacitance take place. They are,

- Transition capacitance (C_T)
- Diffusion capacitance (C_D)

(a) Transition capacitance (C_T):

We know that capacitors store electric charge in the form of electric field. This charge storage is done by using two electrically conducting plates (placed close to each other) separated by an insulating material called dielectric.

The conducting plates or electrodes of the capacitor are good conductors of electricity. Therefore, they easily allow electric current through them. On the other hand, dielectric material or medium is poor conductor of electricity. Therefore, it does not allow electric current through it. However, it efficiently allows electric field.



When voltage is applied to the capacitor, charge carriers start flowing through the conducting wire. When these charge carriers reach the electrodes of the capacitor, they experience a strong opposition from the

dielectric or insulating material. As a result, a large number of charge carriers are trapped at the electrodes of the capacitor. These charge carriers cannot move between the plates. However, they exert an electric field between the plates. The charge carriers which are trapped near the dielectric material will store electric charge. The ability of the material to store electric charge is called capacitance.

In a basic capacitor, the capacitance is directly proportional to the size of electrodes or plates and inversely proportional to the distance between two plates.

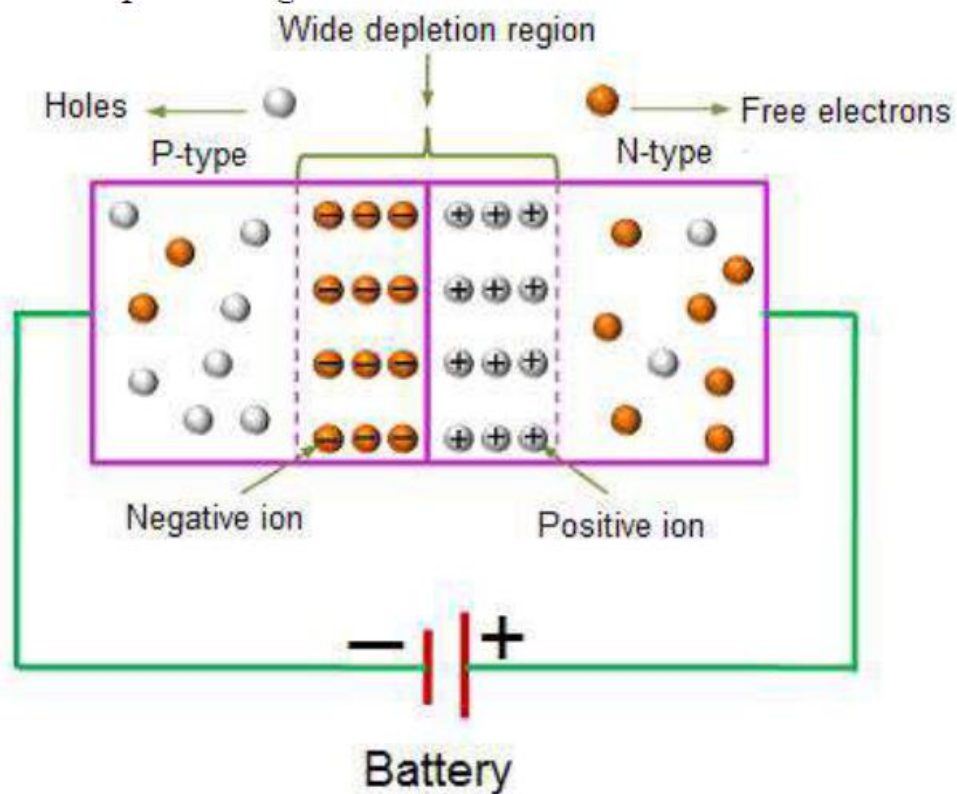
Just like capacitors, a reverse-biased p-n junction diode also stores electric charge in the depletion region. The depletion region is made of immobile positive and negative ions.

In a reverse-biased p-n junction diode, the p-type and n-type regions have low resistance. Hence, p-type and n-type regions act like the electrodes or conducting plates of the capacitor. The depletion region of the p-n junction diode has high resistance. Hence, the depletion region acts like the dielectric or insulating material. Thus, a p-n junction diode can be considered as a parallel plate capacitor.

In the depletion region, the electric charges (positive and negative ions) do not move from one place to another place. However, they exert an electric field or electric force. Therefore, charge is stored in the depletion region in the form of an electric field. The ability of a material to store electric charge is called capacitance. Thus, there exists a capacitance in the depletion region.

The capacitance in the depletion region changes with the change in applied voltage. When reverse bias voltage applied to the p-n junction diode is increased, a large number of holes (majority carriers) from the p-side and electrons (majority carriers) from the n-side are moved away from the p-n junction. As a result, the width of the depletion region increases whereas the size of p-type and n-type regions (plates) decreases.

We know that capacitance means the ability to store electric charge. The p-n junction diode with narrow depletion width and large p-type and n-type regions will store a large amount of electric charge, whereas the p-n junction diode with wide depletion width and small p-type and n-type regions will store only a small amount of electric charge. Therefore, the capacitance of the reverse-biased p-n junction diode decreases when voltage increases.



In a forward biased diode, the transition capacitance exist. However, the transition capacitance is very small compared to the diffusion capacitance. Hence, transition capacitance is neglected in forward biased diode. The amount of capacitance changed with increase in voltage is called transition capacitance. The transition capacitance is also known as depletion region capacitance, junction capacitance or barrier capacitance. Transition capacitance is denoted as C_T .

The change of capacitance at the depletion region can be defined as the change in electric charge per change in voltage.

$$C_T = dQ / dV$$

Where,

C_T = Transition capacitance

dQ = Change in electric charge

dV = Change in voltage

The transition capacitance can be mathematically written as,

$$C_T = \epsilon A / W$$

Where,

ϵ = Permittivity of the semiconductor

A = Area of plates or p-type and n-type regions

W = Width of depletion region

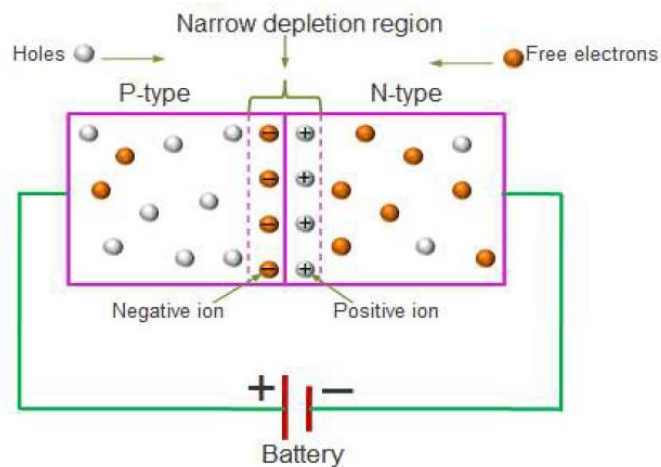
Diffusion capacitance (C_D):

Diffusion capacitance occurs in a forward biased p-n junction diode. Diffusion capacitance is also sometimes referred as storage capacitance. It is denoted as C_D .

In a forward biased diode, diffusion capacitance is much larger than the transition capacitance. Hence, diffusion capacitance is considered in forward biased diode.

The diffusion capacitance occurs due to stored charge of minority electrons and minority holes near the depletion region.

When forward bias voltage is applied to the p-n junction diode, electrons (majority carriers) in the n-region will move into the p-region and recombine with the holes. In the similar way, holes in the p-region will move into the n-region and recombine with electrons. As a result, the width of depletion region decreases.



The electrons (majority carriers) which cross the depletion region and enter into the p-region will become minority carriers of the p-region similarly; the holes (majority carriers) which cross the depletion region and enter into the n-region will become minority carriers of the n-region.

The accumulation of holes in the n-region and electrons in the p-region is separated by a very thin depletion region or depletion layer. This depletion region acts like dielectric or insulator of the capacitor and charge stored at both sides of the depletion layer acts like conducting plates of the capacitor.

Diffusion capacitance is directly proportional to the electric current or applied voltage. If large electric current flows through the diode, a large amount of charge is accumulated near the depletion layer. As a result, large diffusion capacitance occurs.

In the similar way, if small electric current flows through the diode, only a small amount of charge is accumulated near the depletion layer. As a result, small diffusion capacitance occurs.

When the width of depletion region decreases, the diffusion capacitance increases. The diffusion capacitance value will be in the range of nano farads (nF) to micro farads (μF).

The formula for diffusion capacitance is

$$C_D = dQ / dV$$

Where,

C_D = Diffusion capacitance

dQ = Change in number of minority carriers stored outside the depletion region

dV = Change in voltage applied across diode

4.6 HALF WAVE RECTIFIERS

Rectifiers are the circuits used to convert alternating current (AC) into direct current (DC). Half-Wave Rectifiers are designed using a diode (D) and a load resistor (R_L) as shown in Figure 1. In these rectifiers, only one-half of the input waveform is obtained at the output i.e. the output will comprise of either positive pulses or the negative pulses only. The polarity of the output voltage so obtained (across R_L) depends on the direction of the diode used in the circuit of half-wave rectifier. This is evident from the figure as Figure 1a shows the output waveform consisting of only positive pulses while the Figure 1b has only negative pulses in its output waveform.

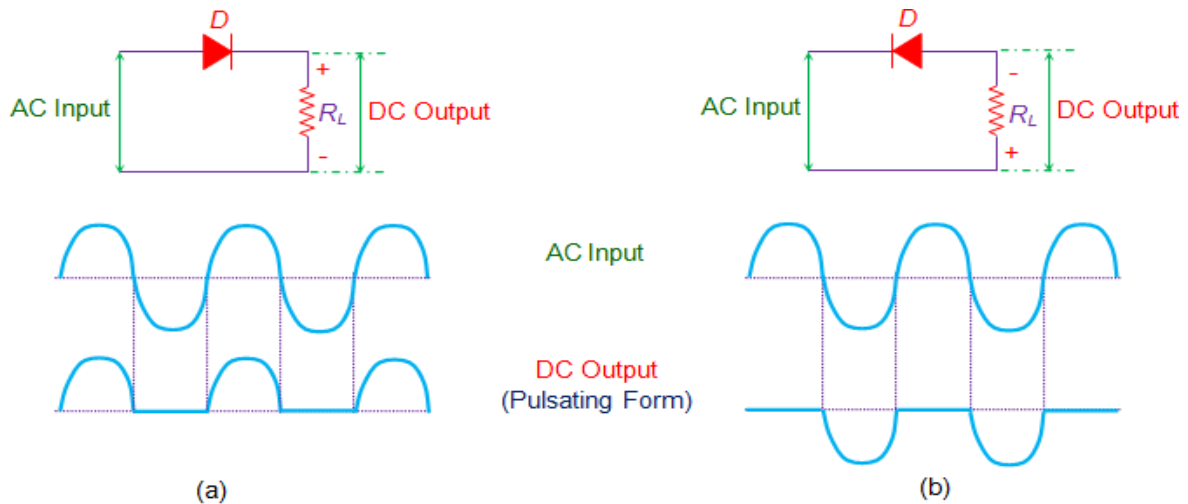


Figure 1 Half Wave Rectifier with Input and Output Waveforms

This is because, in Figure 1a the diode gets forward biased only during the positive pulse of the input which causes the current to flow across R_L , producing the output voltage.

Further for the same case, if the input pulse becomes negative, then the diode will be reverse biased and hence there will be no current flow and no output voltage. Similarly for the circuit shown in Figure 1b, the diode will be forward biased only when the input pulse is negative, and thus the output voltage will contain only the negative pulses. Further it is to be noted that the input to the half-wave rectifier can be supplied even via the transformer. This is advantageous as the transformer provides isolation from the power line as well as helps in obtaining the desired level of DC voltage. Next, one can connect a capacitor across the resistor in the circuit of half wave rectifier to obtain a smoother DC output (Figure 2). Here the capacitor charges through the diode D during the positive pulse of the input while it discharges through the load resistor R_L when the input pulse will be negative. Thus the output waveform of such a rectifier will have ripples in it as shown in the figure.

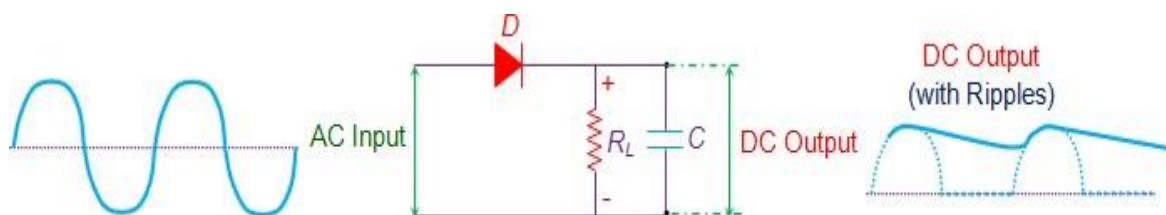


Figure 2 Half-Wave Rectifier with a RC Filter

Different parameters associated with the half wave rectifiers are

1. **Peak Inverse Voltage (PIV):** This is the maximum voltage which should be withstood by the diode under reverse biased condition and is equal to the peak of the input voltage, V_m .
2. **Average Voltage:** This is the DC content of the voltage across the load and is given by V_m/π . Similarly DC current is given as I_m/π , where I_m is the maximum value of the current.
3. **Ripple Factor (r):** It is the ratio of root mean square (rms) value of AC component to the DC component in the output and is given by

$$r = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}$$

Further, for half-wave rectifier, rms voltage is given as $V_m/2$ which results in the ripple factor of 1.21.

4. **Efficiency:** It is the ratio of DC output power to the AC input power and is equal to 40.6 %.
5. **Transformer Utilization Factor:** It is the ratio of DC power delivered to the load to the AC rating of the transformer secondary and is equal to 0.287.
6. **Form Factor:** This is the ratio of rms value to the average value and is thus equal to 1.57 for half- wave rectifier.
7. **Peak Factor:** It is the ratio of peak value to the rms value and is equal to 2.

Half wave rectifiers are advantageous as they are cheap, simple and easy to construct. These are quite rarely used as they have high ripple content in their output. However they can be used in non-critical applications like those of charging the battery. They are also less preferred when compared to other rectifiers as they have low output power, low rectification efficiency and low transformer utilization factor. In addition, if AC input is fed via the transformer, then it might get saturated which in turn results in magnetizing current, hysteresis loss and/or result in the generation of harmonics. Lastly it is important to note that the explanation provided here applies only for the case where the diode is ideal. Although for a practical diode, the basic working remains the same, one will have to consider the voltage drop across the diode as well as its reverse saturation current into consideration during the analysis.

4.7 CENTRE TAP FULLWAVE RECTIFIERS

The circuits which convert the input alternating current (AC) into direct current (DC) are referred to as rectifiers. If such rectifiers rectify both the positive as well as negative pulses of the input waveform, then they are called Full-Wave Rectifiers. Figure 1 shows such a rectifier designed using a multiple winding transformer whose secondary winding is equally divided into two parts with a provision for the connection at its central point (and thus referred to as the centre-tapped transformer), two diodes (D_1 and D_2) and a load resistor (R_L). Here the AC input is fed to the primary winding of the transformer while an arrangement of diodes and the load resistor which yields the DC output, is made across its secondary terminals.

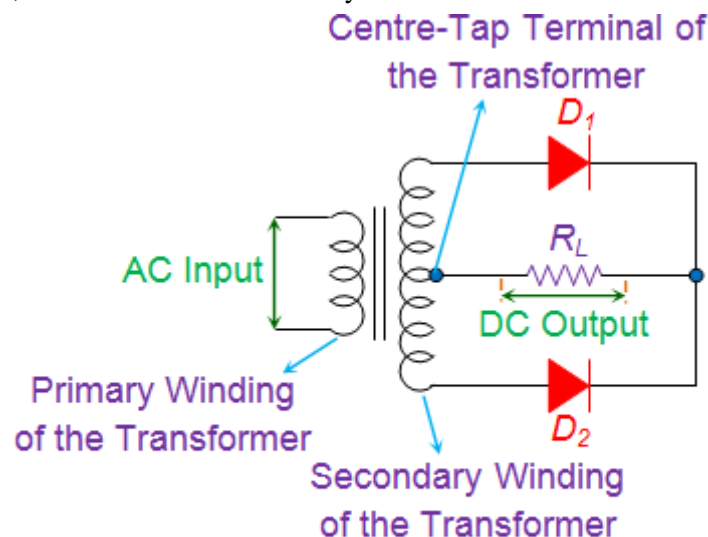


Figure 1 Full Wave Rectifier

The circuit can be analyzed by considering its working during the positive and the negative input pulses separately.

Figure 2a shows the case where the AC pulse is positive in nature i.e. the polarity at the top of the primary winding is positive while its bottom will be negative in polarity. This causes the top part of

the secondary winding to acquire a positive charge while the common centre-tap terminal of the transformer will become negative.

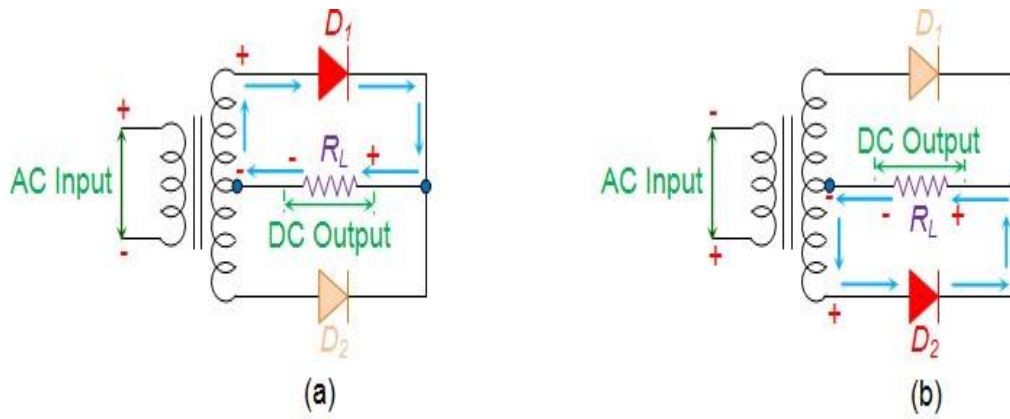


Figure 2 Conduction Path of Full Wave Rectifier for (a) Positive Input Pulse (b) Negative Input Pulse

This causes the diode D_1 to be forward biased which in turn causes the flow of current through R_L along the direction shown in Figure 2a. However at the same time, diode D_2 will be reverse biased and hence acts like an open circuit. This causes the appearance of positive pulse across the R_L , which will be the DC output. Next, if the input pulse becomes negative in nature, then the top and the bottom of the primary winding will acquire the negative and the positive polarities respectively. This causes the bottom of the secondary winding to become positive while its centre-tapped terminal will become negative. Thus the diode D_2 gets forward biased while the D_1 will get reverse biased which allows the flow of current as shown in the Figure 2b. Here the most important thing to note is the fact that the direction in which the current flows via R_L will be identical in either case (both for positive as well as for negative input pulses). Thus we get the positive output pulse even for the case of negative input pulse (Figure 3), which indicates that both the half cycles of the input AC are rectified.

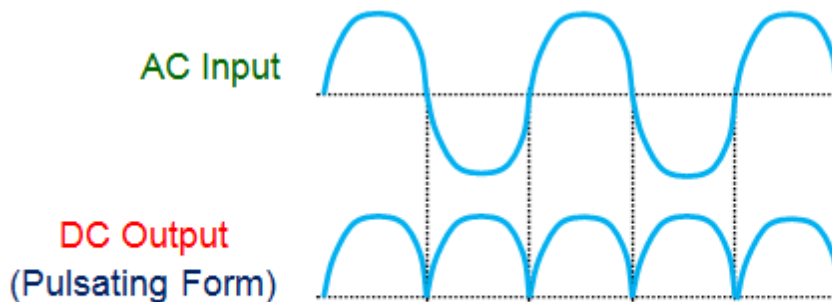


Figure 3 Input and Output Waveforms of Full Wave Rectifier

Such circuits are referred to as (i) Centre-Tapped **Full Wave Rectifiers** as they use a centre-tapped transformer, (ii) Two-Diode Full-Wave Rectifiers because of the use of two diodes and/or (iii) Bi-Phase Circuits due to the fact that in these circuits, the output voltage will be the phasor addition of the voltages developed across the load resistor due to two individual diodes, where each of them conducts only for a particular half-cycle. However as evident from Figure 3, the output of the rectifier is not pure DC but pulsating in nature, where the frequency of the output waveform is seen to be double of that at the input. In order to smoothen this, one can connect a capacitor across the load resistor as shown by the Figure 4. This causes the capacitor to charge via the diode D_1 as long as the input positive pulse increases in its magnitude. By the time the input pulse reaches the positive maxima, the capacitor would have charged to the same magnitude. Next, as long as the input positive pulse keeps

decreasing, the capacitor tries to hold the charge acquired (being an energy-storage element).

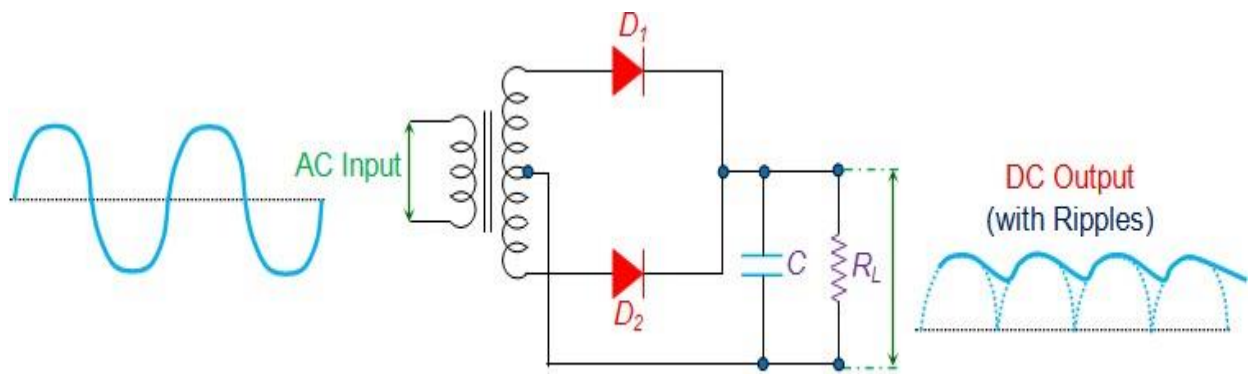


Figure 4 Full-Wave Rectifier with a RC Filter

However there will be voltage-loss as some amount of charge gets lost through the path provided by the load resistor (nothing but discharging phenomenon). Further, as the input pulse starts to go low to reach the negative maxima, the capacitor again starts to charge via the path provided by the diode D_2 and acquires an almost equal voltage but with opposite polarity. Next, as the input voltage starts to move towards 0V, the capacitor slightly discharges via R_L . This charge-discharge cycle of the capacitor causes the ripples to appear in the output waveform of the full-wave rectifier with RC filter as shown in Figure 4.

Different parameters and their values for the centre-tapped full-wave rectifiers are

1. **Peak Inverse Voltage (PIV):** This is the maximum voltage which occurs across the diodes when they are reverse biased. Here it will be equal to twice the peak of the input voltage, $2V_m$.
2. **Average Voltage:** It is the DC voltage available across the load and is equal to $2V_m/\pi$. The corresponding DC current will be $2I_m/\pi$, where I_m is the maximum value of the current.
3. **Ripple Factor (r):** This is the ratio of the root mean square (rms) value of AC component to the dc component at the output. It is given by and will be equal to 0.482 as the rms voltage for a full-wave rectifier is given as

$$r = \frac{\sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}}{V_m / \sqrt{2}}$$

4. **Efficiency:** This is the ratio of DC output power to the AC input power and is equal to 81.2 %.
5. **Transformer Utilization Factor (TUF):** This factor is expressed as the ratio of DC power delivered to the load to the AC rating of the transformer secondary. For the full-wave rectifier this will be 0.693.
6. **Form Factor:** This is the ratio of rms value to the average value and is equal to 1.11.
7. **Peak Factor:** It is the ratio of peak value to the rms value and is equal to $\sqrt{2}$ for the full-wave rectifiers.

Further it is to be noted that the two-diode full-wave rectifier shown in Figure 1 is costly and bulky in size as it uses the complex centre-tapped transformer in its design. Thus one may resort to another type of full-wave rectifier called Full-Wave Bridge Rectifier (identical to Bridge Rectifier) which might or might not involve the transformer (even if used, will not be as complicated as a centre-tap one). It also offers higher TUF and higher PIV which makes it ideal for high power applications. However it is to be noted that the full wave bridge rectifier uses four diodes instead of two, which in turn increases the magnitude of voltage drop across the diodes, increasing the heating loss. **Full wave rectifiers** are used in general power supplies, to charge a battery and to provide power to the devices like motors, LEDs, etc. However due to the ripple content in the output waveform, they are not

preferred for audio applications. Further these are advantageous when compared to half-wave rectifiers as they have higher DC output power, higher transformer utilization factor and lower ripple content, which can be made more smoother by using π -filters. All these merits mask-up its demerit of being costly in comparison to the half-wave rectifiers due to the use of increased circuit elements. At last, it is to be noted that the explanation provided here considers the diodes to be ideal in nature. So, incase of practical diodes, one will have to consider the voltage drop across the diode, its reverse saturation current and other diode characteristics into account and reanalyze the circuit. Nevertheless the basic working remains the same.

4.10 FULLWAVE BRIDGE RECTIFIERS

Bridge Rectifiers are the circuits which convert alternating current (AC) into direct current (DC) using the diodes arranged in the bridge circuit configuration. They usually comprise of four or more number of diodes which cause the output generated to be of the same polarity irrespective of the polarity at the input. Figure 1 shows such a bridge rectifier composed of four diodes D_1 , D_2 , D_3 and D_4 in which the input is supplied across two terminals A and B in the figure while the output is collected across the load resistor R_L connected between the terminals C and D.

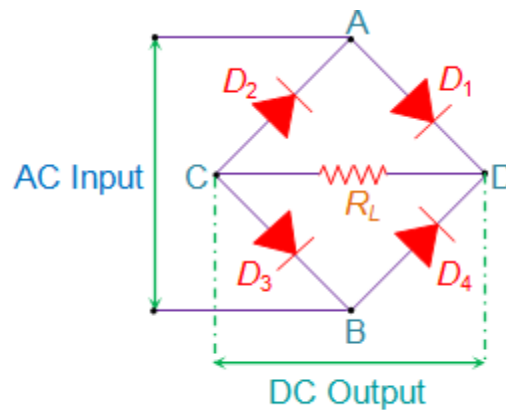


Figure 1 Bridge Rectifier

Now consider the case wherein the positive pulse appears at the AC input i.e. the terminal A is positive while the terminal B is negative. This causes the diodes D_1 and D_3 to get forward biased and at the same time, the diodes D_2 and D_4 will be reverse biased.

As a result, the current flows along the short-circuited path created by the diodes D_1 and D_3 (considering the diodes to be ideal), as shown by Figure 2a. Thus the voltage developed across the load resistor R_L will be positive towards the end connected to terminal D and negative at the end connected to the terminal C.

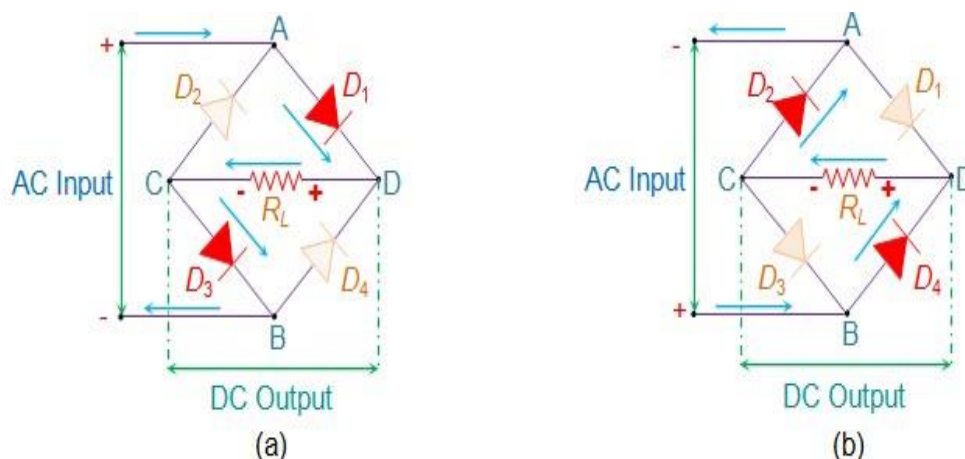


Figure 2 Current Path Through the Bridge Rectifier for (a) Positive half-cycle (b) Negative Half-Cycle

Next if the negative pulse appears at the AC input, then the terminals A and B are negative and positive respectively. This forward biases the diodes D_2 and D_4 , while reverse biasing D_1 and D_3 which causes the current to flow in the direction shown by Figure 2b. At this instant, one has to note that the polarity of the voltage developed across R_L is identical to that produced when the incoming AC pulse was positive in nature. This means that for both positive and negative pulse, the output of the bridge rectifier will be identical in polarity as shown by the wave forms in Figure 3.

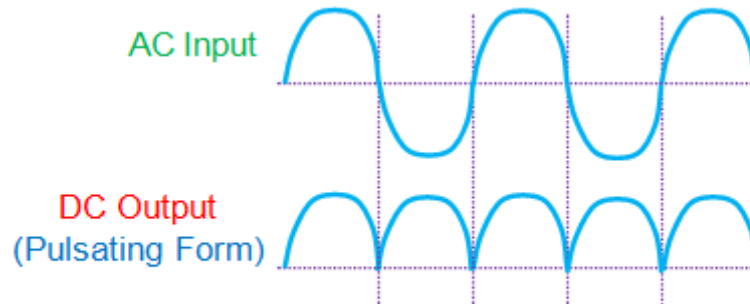


Figure 3 Input-Output Waveforms of a Bridge Rectifier

However it is to be noted that the bridge rectifier's DC will be pulsating in nature. In order to obtain pure form of DC, one has to use capacitor in conjunction with the bridge circuit (Figure 4).

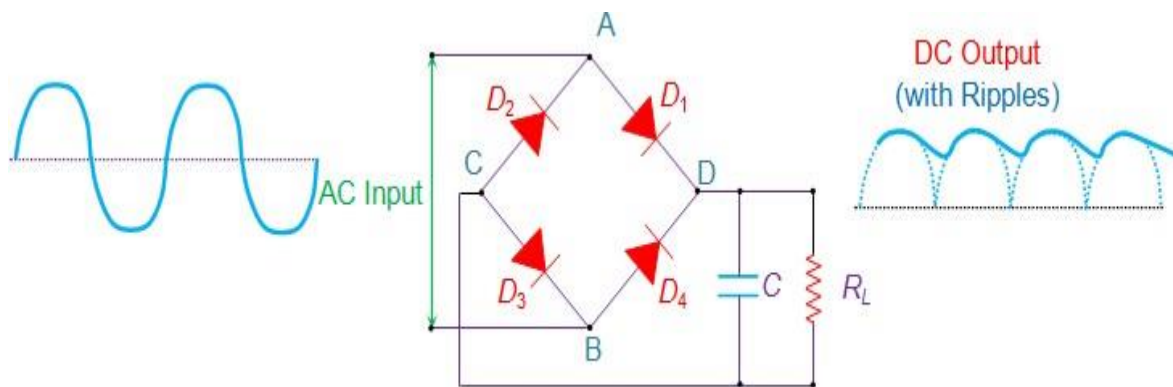


Figure 4 Bridge Rectifier with a RC Filter

In this design, the positive pulse at the input causes the capacitor to charge through the diodes D_1 and D_3 . However as the negative pulse arrives at the input, the charging action of the capacitor ceases and it starts to discharge via R_L . This results in the generation of DC output which will have ripples in it as shown in the figure. This ripple factor is defined as the ratio of AC component to the DC component in the output voltage. In addition, the mathematical expression for the ripple voltage is given by the equation

$$V_r = \frac{I_l}{fC}$$

Where, V_r represents the ripple voltage.

I_l represents the load current.

f represents the frequency of the ripple which will be twice the input frequency. C is the Capacitance.

Further, the **bridge rectifiers** can be majorly of two types, viz., Single-Phase Rectifiers and Three-Phase Rectifiers. In addition, each of these can be either Uncontrolled or Half-Controlled or Full-Controlled. Bridge rectifiers for a particular application are selected by considering the load current requirements. These bridge rectifiers are quite advantageous as they can be constructed with or without a transformer and are suitable for high voltage applications. However here two diodes will be conducting for every half-cycle and thus the voltage drop across the diodes will be higher. Lastly one has to note that apart from converting AC to DC, **bridge rectifiers** are also used to detect the amplitude of modulated radio signals and to supply polarized voltage for welding applications.

3.23.1. Filters Circuit

An electronic circuit or device which blocks the a.c. components but allows the d.c. components of the rectifiers to pass to the load is called a filter circuit.

Types of filter circuit:-

- (i) Shunt Capacitor Filter
- (ii) Series Inductor Filter
- (iii) Choke Input (LC) Filter
- (iv) Capacitor Input (π) Filter
- (i) **Shunt Capacitor Filter**

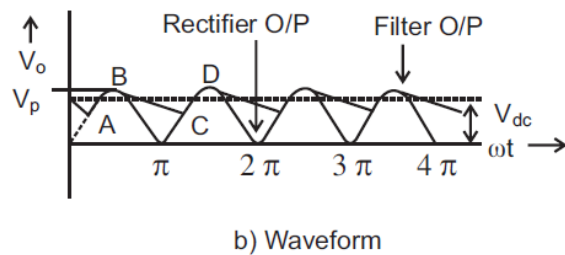
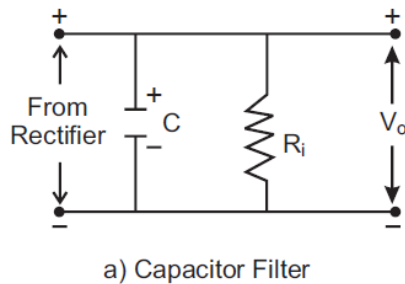


Figure-18

Working

The working of a shunt capacitor filter can be explained with the help of a wave diagram shown in fig.18 the dotted pulsating wave shows the output of a full wave rectifier. When the rectifier voltage is increasing the capacitor is charged to $+V_m$. at point b the rectifier voltage tries to fall but the charged capacitor immediately tries to send the current back to rectifier. In the process the rectifier diodes are reverse biased and stop giving supply to the load. Thus the capacitor discharges (B to C) through the load. The capacitor continues to discharge until the source voltage becomes more than the capacitor voltage. The diode again starts conducting and the capacitor is again charged to peak value $+V_m$ (point). During this time the rectifier supplies the charging current I and the load current.

From above it is clear that capacitor not only remove the a.c. component but also improves the output voltage. The smoothness and magnitude of output voltage depends upon the time constant CR . The longer the time period the steadier is the output voltage. This can be achieved by using a large value of capacitor.

However the maximum value of the capacitance that can be employed is limited by the current that can be safely handled by the diode. The diodes employed in the rectifier circuit can deliver maximum current as per their rating. Therefore the size of the capacitor has to be limited so that it may not draw current more than the rating of the diodes.

(ii) Series Inductor Filter

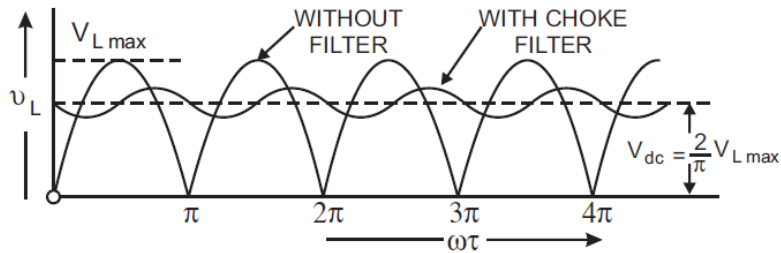
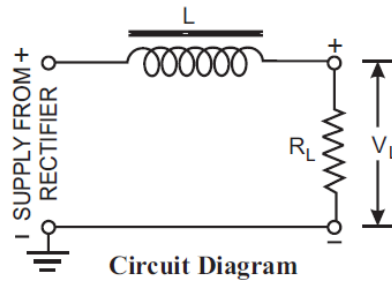


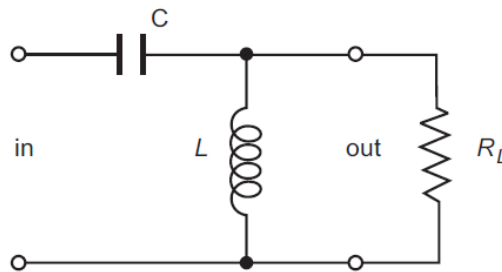
Figure-19

A series inductor filter is shown. In this case an inductor is just connected in series with load. The inductor has the inherent property to oppose the change of current. This property of inductor utilised here to suppress the a.c. component (ripples) from the output of the rectifier.

The reactance ($X=2\pi fL$) of the inductor is large for high frequencies and offers more opposition to them but it allows the d.c. component of the rectifier output. Hence an inductance blocks the a.c. components but allows the d.c. components to reach the load. Thus it smooths out the rectifier output as shown fig.-19.

(iii) Choke Input LC Filter

A choke input LC filter is shown fig.-20. In this case an inductor is connected in series and a capacitor is parallel with the load.

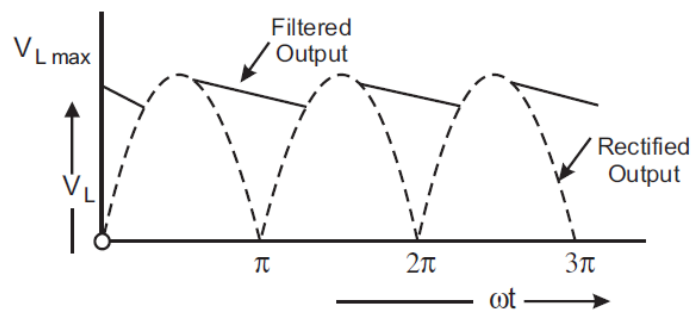
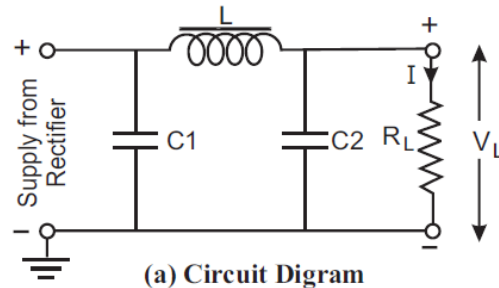


The output of a full wave rectifier contains a.c. components of a fundamental frequencies 100 Hz. The inductor offers a high opposition to the a.c component and blocks it but allows the d.c. component to pass through the low reactance of the capacitor. Hence almost pure d.c. reaches at

the load. Although the output of this filter is almost d.c. but still it contains small a.c. component. To improve it further one or more sets of LC filter may be applied further.

(iv) Capacitor Input (PIE) Filter

A capacitor input filter is shown fig.-21(a). In this case an additional capacitor C , is connected in the beginning across the output terminals of the rectifier. Since its shape is like the Greek letter (PIE) it is named as pie rectifier.



(b) Rectified and Filtered Output Voltage Waveform Full-Wave Rectifier with Capacitor Input Filter
Figure-21

The filter action of three components $C1$, L and $C2$ is described below:

- (I) **Action of $C1$:** It provides an easy path to the a.c. components and by pass it and blocks d.c. components which continues its journey through the inductor choke. It also increases the magnitude of V_{av} because of its charging and discharging action.
- (II) **Action of L :** It provides an easy path to d.c. component but blocks the a.c. components because of its high reactance.
- (III) **Action of $C2$:** Any a.c component which the inductor has failed to block is by passed by this capacitor and only pure d.c. appears across the load.

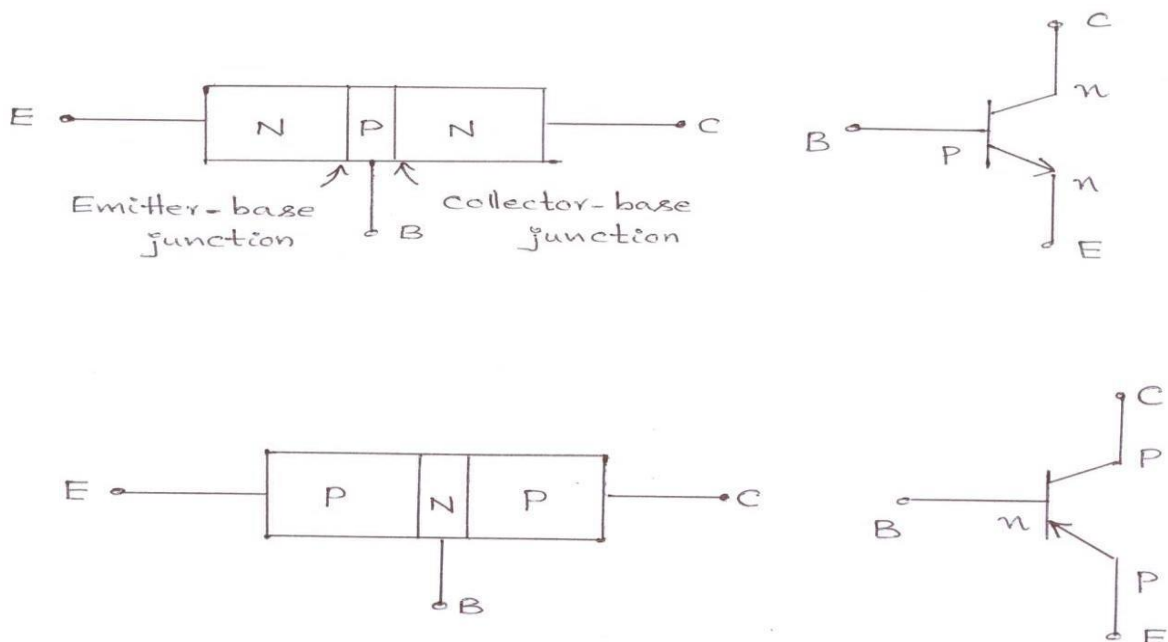
UNIT V

BIPOLAR JUNCTION TRANSISTOR AND JFET

5.1 INTRODUCTION

- The transistor was invented in 1947 by John Bardeen, Walter Brattain and William Shockley at Bell Laboratory in America. A transistor is a semiconductor device, commonly used as an Amplifier or an electrically Controlled Switch.
- There are two types of transistors:
 - 1) Unipolar Junction Transistor
 - 2) Bipolar Junction Transistor
- In Unipolar transistor, the current conduction is only due to one type of carriers i.e., majority charge carriers. The current conduction in bipolar transistor is because of both the types of charge carriers i.e., holes and electrons. Hence it is called as Bipolar Junction Transistor and it is referred to as BJT.
- BJT is a semiconductor device in which one type of semiconductor material is sandwiched between two opposite types of semiconductor i.e., an n-type semiconductor is sandwiched between two p-type semiconductors or a p-type semiconductor is sandwiched between two n-type semiconductor.
- Hence the BJTs are of two types. They are:
 - 1) n-p-n Transistor
 - 2) p-n-p Transistor

The two types of BJTs are shown in the figure below.



- The arrow head represents the conventional current direction from p to n. Transistor has three terminals.
 - 1) Emitter
 - 2) Base
 - 3) Collector

➤ Transistor has two p-n junctions. They are:

- 1) Emitter-Base Junction
- 2) Collector-Base Junction

Emitter: Emitter is heavily doped because it is to emit the charge carriers.

Base: The charge carriers emitted by the emitter should reach collector passing through the base. Hence base should be very thin and to avoid recombination, and to provide more collector current base is lightly doped.

Collector: Collector has to collect the most of charge carriers emitted by the emitter. Hence the area of cross section of collector is more compared to emitter and it is moderately doped.

Transistor can be operated in three regions.

- 1) Active region.
- 2) Saturation region.
- 3) Cut-Off region.

Active Region: For the transistor to operate in active region base to emitter junction is forward biased and collector to base junction is reverse biased.

Saturation Region: Transistor to be operated in saturation region if both the junctions i.e., collector to base junction and base to emitter junction are forward biased.

Cut-Off Region: For the transistor to operate in cut-off region both the junctions i.e., base to emitter junction and collector to base junction are reverse biased.

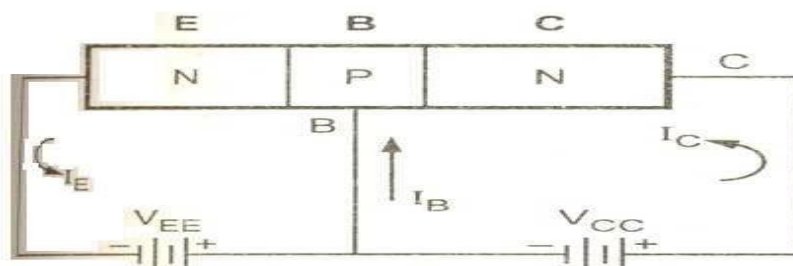
Transistor can be used as

- 1) Amplifier
- 2) Switch

For the transistor to act as an amplifier, it should be operated in active region. For the transistor to act as a switch, it should be operated in saturation region for ON state, and cut-off region for OFF state.

5.2 TRANSISTOR OPERATION:

Working of a N-P-N Transistor:

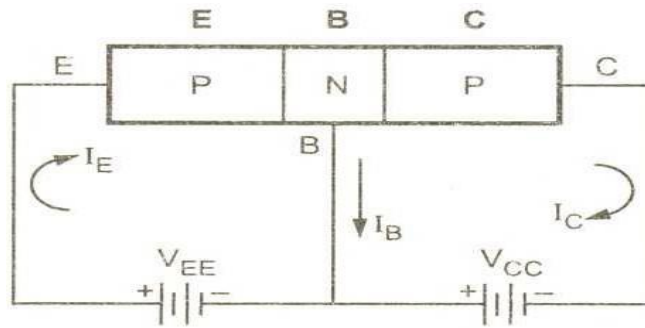


The n-p-n transistor with base to emitter junction forward biased and collector base junction reverse biased is as shown in figure.

As the base to emitter junction is forward biased the majority carriers emitted by the n type emitter i.e., electrons have a tendency to flow towards the base which constitutes the emitter Current I_E . As the base is p-type there is chance of recombination of electrons emitted by the emitter with the holes in the p-type base. But as the base is very thin and lightly doped only few electrons emitted by the n-type emitter less than 5% combines with the holes in the p-type base, the remaining more than 95% electrons emitted by the n-type emitter cross over into the collector region constitute the collector current.

The current distributions are as shown in fig $I_E = I_B + I_C$

Working of a P-N-P Transistor:



The p-n-p transistor with base to emitter junction is forward biased and collector to base junction reverse biased is as show in figure. As the base to emitter junction is forward biased the majority carriers emitted by the type emitter i.e., holes have a tendency to flow towards the base which constitutes the emitter current I_E . As the base is n-type there is a chance of recombination of holes emitted by the emitter with the electrons in the n-type base. But as the base us very thin and lightly doped only few electrons less than 5% combine with the holes emitted by the p-type emitter, theremaining 95% charge carriers cross over into the collector region to constitute the collector current. The current distributions are shown in figure.

$$I_E = I_B + I_C$$

5.1 TRANSISTOR CIRCUIT CONFIGURATIONS:

Following are the three types of transistor circuit configurations:

- 1) Common-Base (CB)
- 2) Common-Emitter (CE)
- 3) Common-Collector (CC)

Here the term 'Common' is used to denote the transistor lead which is common to the input and output circuits. The common terminal is generally grounded. It should be remembered that regardless the circuit configuration, the emitter is always forward-biased while the collector is always reverse-biased.

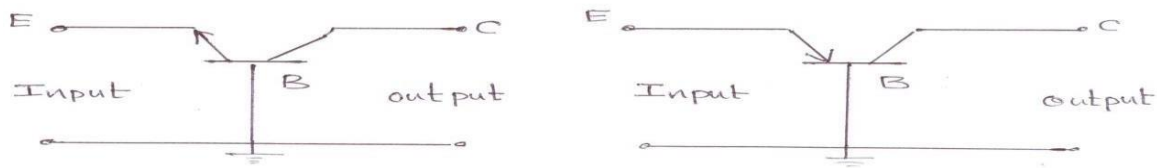


Fig. Common – Base Configuration

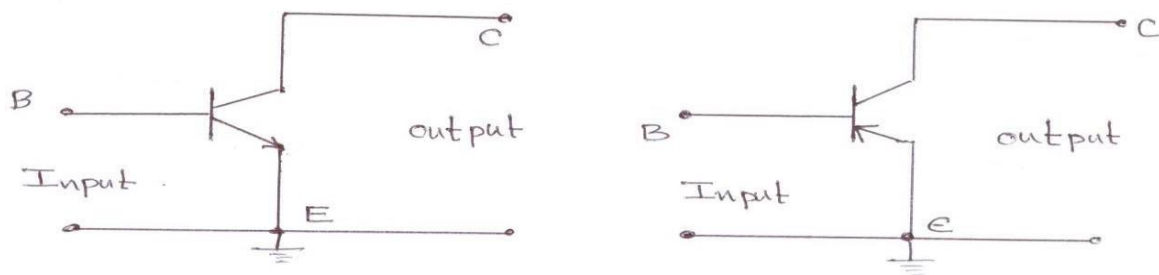


Fig. Common – Emitter Configuration

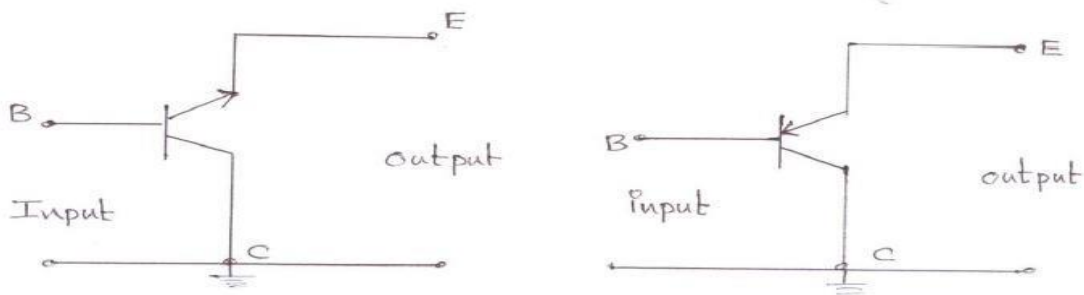


Fig. Common – Collector Configuration

5.1.1 Common – Base (CB) Configurations:

In this configuration, the input signal is applied between emitter and base while the output is taken from collector and base. As base is common to input and output circuits, hence the name common-base configuration. Figure show the common-base P-N-P transistor circuit.

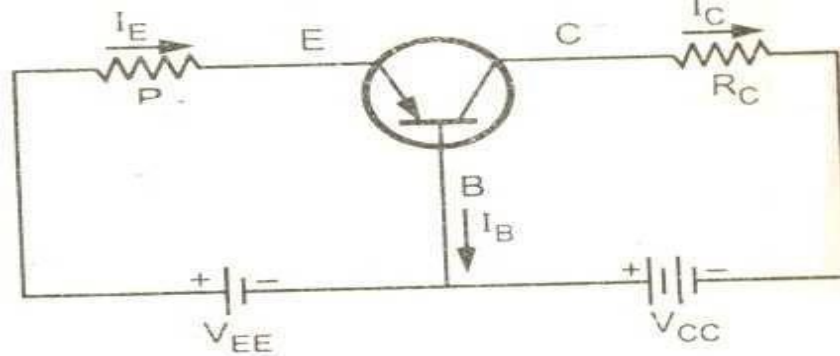


Fig. Common – Base PNP transistor amplifier.

Current Amplification Factor (α) :

When no signal is applied, then the ratio of the collector current to the emitter current is called dc alpha (α_{dc}) of a transistor.

$$\alpha_{dc} = \frac{\Delta I_C}{\Delta I_E}$$

(Negative sign signifies that I_E flows into transistor while I_C flows out of it). ‘ α ’ of a transistor is a measure of the quality of a transistor. Higher is the value of ‘ α ’, better is the transistor in the sense that collector current approaches the emitter current. By considering only magnitudes of the currents, $I_C = \alpha I_E$ and hence $I_B = I_E - I_C$

Therefore,

$$I_B = I_E - \alpha I_E = I_E(1 - \alpha)$$

For all practical purposes, $\alpha_{dc} = \alpha_{ac} = \alpha$ and practical values in commercial transistors range from 0.9 to 0.99.

Total Collector Current:

The total collector current consists of the following two parts

- i) I_E current due to majority carriers
- ii) I_{CBO} current due to minority carriers

$$\text{Total collector current } I_C = \alpha I_E + I_{CBO}$$

The collector current can also be expressed as $I_C = \alpha (I_B + I_C) + I_{CBO}$ (Q $I_E = I_B + I_C$)

$$\Rightarrow I_C(1 - \alpha) = \alpha I_B + I_{CBO}$$

$$\Rightarrow I_C = \left(\frac{\alpha}{1 - \alpha}\right) I_B + \left(\frac{1}{1 - \alpha}\right) I_{CBO}$$

5.1.2 COMMON-EMITTER (CE) CONFIGURATION:

In this configuration, the input signal is applied between base and emitter and the output is taken from collector and emitter. As emitter is common to input and output circuits, hence the name common emitter configuration.

Figure shows the Common-Emitter P-N-P transistor circuit.

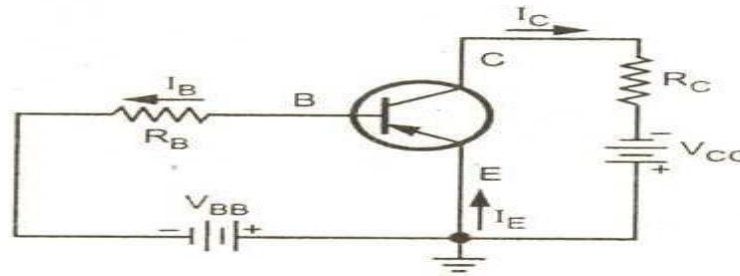


Fig. Common-Emitter PNP transistor amplifier.

Current Amplification Factor (β):

When no signal is applied, then the ratio of collector current to the base current is called dc beta (β_{dc}) of a transistor.

$$\beta_{dc} = \beta = \frac{I_C}{I_B} \dots\dots\dots(1)$$

When signal is applied, the ratio of change in collector current to the change in base current is defined as base current amplification factor. Thus,

$$\beta_{dc} = \beta = \frac{\Delta I_C}{\Delta I_B} \dots\dots\dots(2)$$

From equation (1), $I_C = \beta I_B$

Almost in all transistors, the base current is less than 5% of the emitter current. Due to this fact, ' β ' ranges from 20 to 500. Hence this configuration is frequently used when appreciable current gain as well as voltage gain is required.

Total Collector Current:

The Total collector current $I_C = \beta I_B + I_{CEO} \dots\dots\dots(3)$

Where I_{CEO} is the leakage current.

But, we have, $I_C = \left(\frac{\alpha}{1-\alpha}\right)I_B + \left(\frac{1}{1-\alpha}\right)I_{CBO} \dots\dots\dots(4)$

Comparing equations (3) and (4), we get

$$\beta = \frac{\alpha}{1-\alpha} \text{ and } I_{CEO} = \frac{1}{1-\alpha} I_{CBO} \dots\dots\dots(5)$$

Relation between α and β :

We know that $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$

$$I_E = I_B + I_C \quad (\text{or}) \quad I_B = I_E - I_C$$

Now
$$\beta = \frac{I_C}{I_E - I_C} = \frac{\frac{I_C}{I_E}}{1 - \frac{I_C}{I_E}} = \frac{\alpha}{1-\alpha} \dots\dots\dots(6)$$

$$\Rightarrow \beta(1-\alpha) = \alpha \quad (\text{or}) \quad \beta = \alpha(1+\beta)$$

$$\Rightarrow \alpha = \frac{\beta}{1+\beta} \dots\dots\dots(7)$$

It can be seen that $1-\alpha = \frac{1}{1+\beta} \dots\dots\dots(8)$

5.1.3 COMMON – COLLECTOR (CC) CONFIGURATION:

In this configuration, the input signal is applied between base and collector and the output is taken from the emitter. As collector is common to input and output circuits, hence the name common collector configuration. Figure shows the common collector PNP transistor circuit.

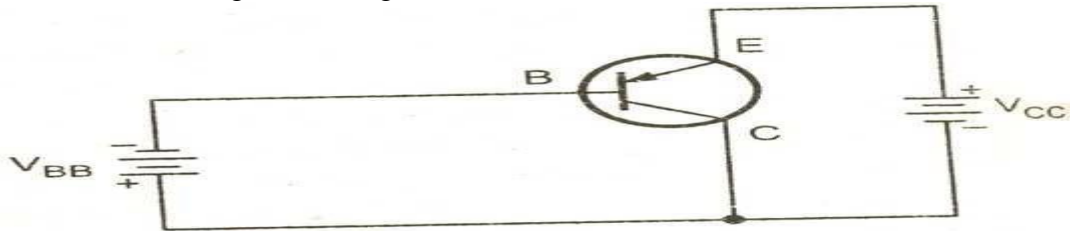


Fig. Common Collector PNP transistor amplifier.

Current Amplification Factor (γ):

When no signal is applied, then the ratio of emitter current to the base current is called as dc gamma (γ_{dc}) of the transistor.

$$\gamma_{dc} = \gamma = \frac{I_E}{I_B} \quad \dots\dots\dots (1)$$

5.4 CHARACTERISTICS OF COMMON-BASE CIRCUIT:

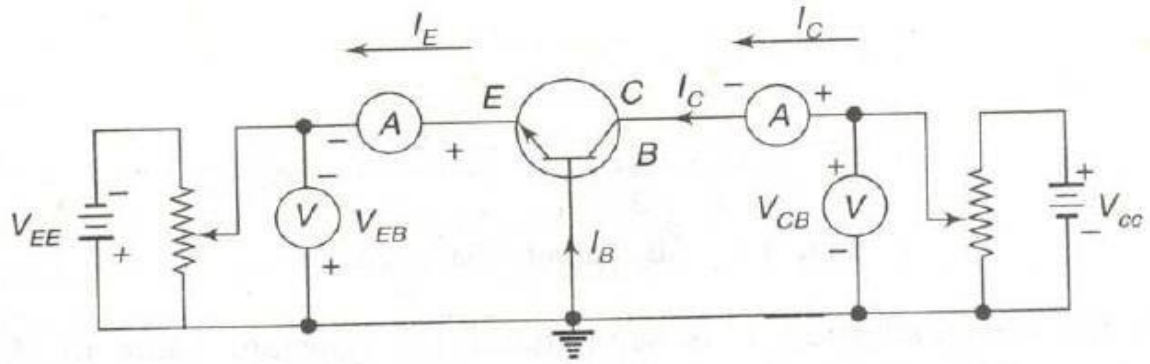


Fig. Circuit to determine CB static characteristics.

Input Characteristics:

To determine the input characteristics, the collector-base voltage V_{CB} is kept constant at zero volts and the emitter current I_E is increased from zero in suitable equal steps by increasing V_{EB} . This is repeated for higher fixed values of V_{CB} . A curve is drawn between emitter current I_E and emitter-base voltage V_{EB} at constant collector-base voltage V_{CB} .

The input characteristics thus obtained are shown in figure below.

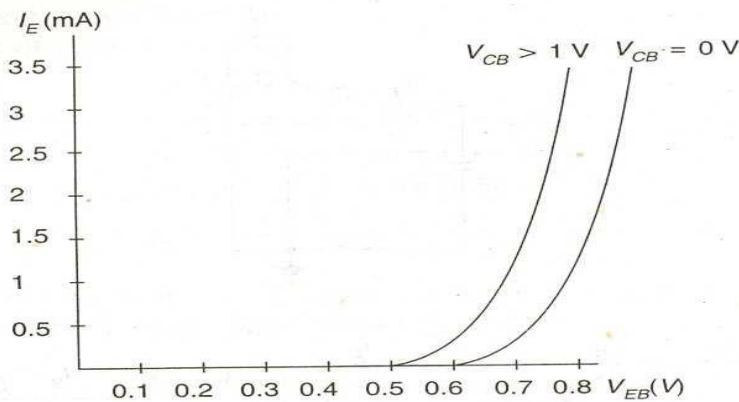


Fig. CB Input Characteristics.

Output Characteristics:

To determine the output characteristics, the emitter current I_E is kept constant at a suitable value by adjusting the emitter-base voltage V_{EB} . Then V_{CB} is increased in suitable equal steps and the collector current I_C is noted for each value of I_E . Now the curves of I_C versus V_{CB} are plotted for constant values of I_E and the output characteristics thus obtained is shown in figure below.

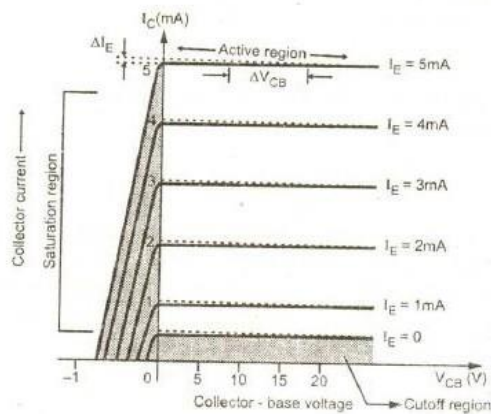


Fig. CB Output Characteristics

From the characteristics, it is seen that for a constant value of I_E , I_C is independent of V_{CB} and the curves are parallel to the axis of V_{CB} . Further, I_C flows even when V_{CB} is equal to zero. As the emitter-base junction is forward biased, the majority carriers, i.e., electrons, from the emitter are injected into the base region. Due to the action of the internal potential barrier at the reverse biased collector-base junction, they flow to the collector region and give rise to I_C even when V_{CB} is equal to zero.

It is the slope of CB output characteristics I_C versus V_{CB} .

5.5 CHARACTERISTICS OF COMMON-EMITTER CIRCUIT:

The circuit diagram for determining the static characteristic curves of the an N-P-N transistor in the common emitter configuration is shown in figure below.

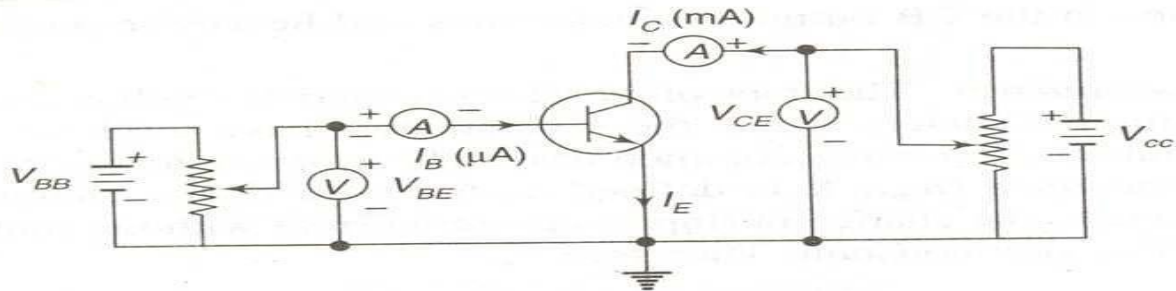


Fig. Circuit to determine CE Static characteristics.

Input Characteristics:

To determine the input characteristics, the collector to emitter voltage is kept constant at zero volts and base current is increased from zero in equal steps by increasing V_{BE} in the circuit. The value of V_{BE} is noted for each setting of I_B . This procedure is repeated for higher fixed values of V_{CE} , and the curves of I_B versus V_{BE} are drawn.

The input characteristics thus obtained are shown in figure below.

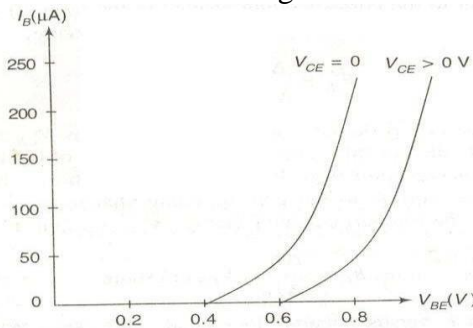


Fig. CE Input Characteristics.

When $V_{CE}=0$, the emitter-base junction is forward biased and the junction behaves as a forward biased diode. When V_{CE} is increased, the width of the depletion region at the reverse biased collector-

base junction will increase. Hence the effective width of the base will decrease. This effect causes a decrease in the base current I_B . Hence, to get the same value of I_B as that for $V_{CE}=0$, V_{BE} should be increased. Therefore, the curve shifts to the right as V_{CE} increases.

Output Characteristics:

To determine the output characteristics, the base current I_B is kept constant at a suitable value by adjusting base-emitter voltage, V_{BE} . The magnitude of collector-emitter voltage V_{CE} is increased in suitable equal steps from zero and the collector current I_C is noted for each setting of V_{CE} . Now the curves of I_C versus V_{CE} are plotted for different constant values of I_B . The output characteristics thus obtained are shown in figure below.

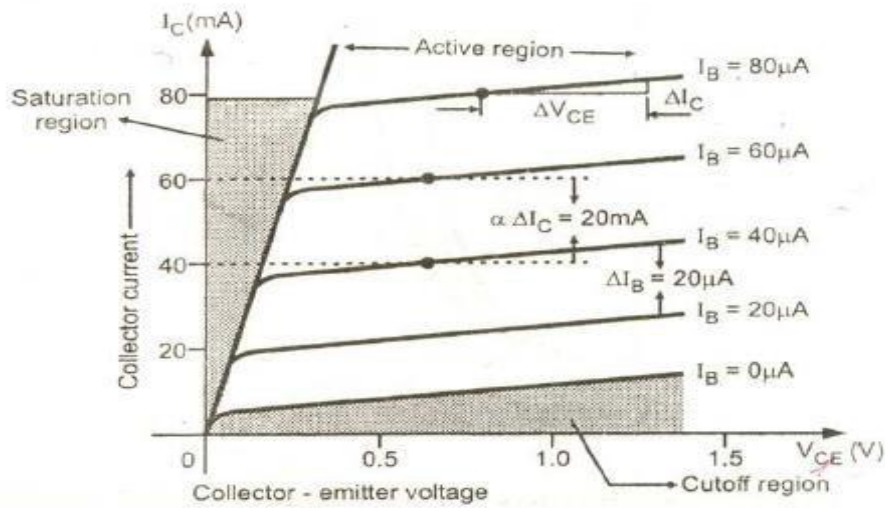


Fig. CE Output characteristics

The output characteristics of common emitter configuration consist of three regions: Active, Saturation and Cut-off regions.

Active Region:

The region where the curves are approximately horizontal is the “Active” region of the CE configuration. In the active region, the collector junction is reverse biased. As V_{CE} is increased, reverse bias increase. This causes depletion region to spread more in base than in collector, reducing the changes of recombination in the base. This increase the value of a dc . This Early effect causes collector current to rise more sharply with increasing V_{CE} in the active region of output characteristics of CE transistor.

Saturation Region:

If V_{CE} is reduced to a small value such as 0.2V, then collector-base junction becomes forward biased, since the emitter-base junction is already forward biased by 0.7V. The input junction in CE configuration is base to emitter junction, which is always forward biased to operate transistor in active region. Thus input characteristics of CE configuration are similar to forward characteristics of p-n junction diode. When both the junctions are forwards biased, the transistor operates in the saturation region, which is indicated on the output characteristics. The saturation value of V_{CE} , designated $V_{CE}(\text{Sat})$, usually ranges between 0.1V to 0.3V.

Cut-Off Region:

When the input base current is made equal to zero, the collector current is the reverse leakage current I_{CEO} . Accordingly, in order to cut off the transistor, it is not enough to reduce $I_B=0$. Instead, it is necessary to reverse bias the emitter junction slightly. We shall define cut off as the condition where the collector current is equal to the reverse saturation current I_{CO} and the emitter current is zero.

5.5 Characteristics of common collector circuit:

The circuit diagram for determining the static characteristics of an N-P-N transistor in the common collector configuration is shown in fig. below.

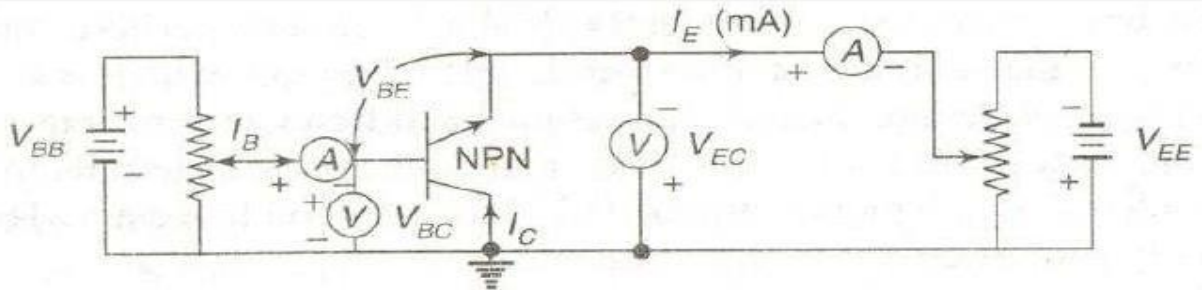


Fig. Circuit to determine CC static characteristics.

Input Characteristics:

To determine the input characteristic, V_{EC} is kept at a suitable fixed value. The base collector voltage V_{BC} is increased in equal steps and the corresponding increase in I_B is noted. This is repeated for different fixed values of V_{EC} . Plots of V_{BC} versus I_B for different values of V_{EC} shown in figure are the input characteristics.

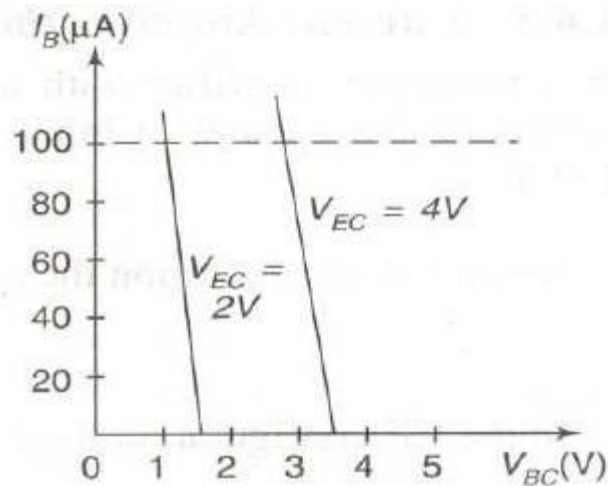


Fig. CC Input Characteristics.

Output Characteristics:

The output characteristics shown in figure below are the same as those of the common emitter configuration.

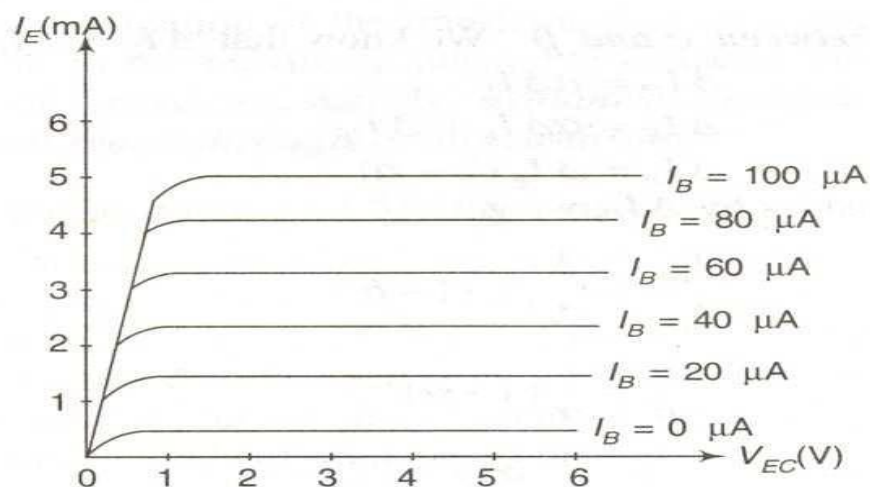
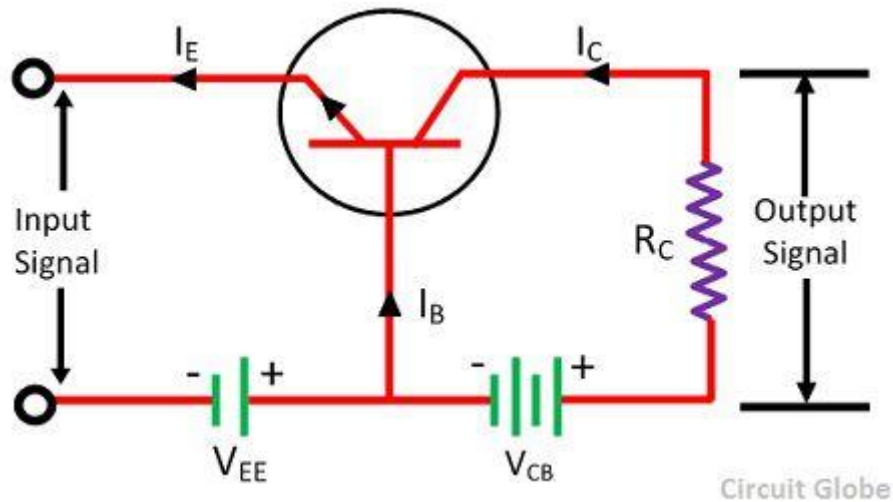


Fig. CC output characteristics.

Transistor as an Amplifier :

The transistor raises the strength of a weak signal and hence acts as an amplifier. The transistor amplifier circuit is shown in the figure below. The transistor has three terminals namely emitter, base and collector. The emitter and base of the transistor are connected in forward bias and the collector base region is in reverse bias. The forward bias means the P-region of the transistor is connected to the positive terminal of the supply and the negative region is connected to the N-terminal and in reverse bias just opposite of it has occurred.



The input signal or weak signal is applied across the emitter base and the output is obtained to the load resistor R_C which is connected in the collector circuit. The DC voltage V_{EE} is applied to the input circuit along with the input signal to achieve the amplification. The DC voltage V_{EE} keeps the emitter-base junction under the forward biased condition regardless of the polarity of the input signal and is known as a bias voltage.

In the collector circuit, a load resistor R_C of high value is connected. When collector current flows through such a high resistance, it produces a large voltage drop across it. Thus, a weak signal (0.1V) applied to the input circuit appears in the amplified form (10V) in the collector circuit.

FIELD EFFECT TRANSISTOR

INTRODUCTION

1. The Field effect transistor is abbreviated as FET , it is an another semiconductor device like a BJT which can be used as an amplifier or switch.
2. The Field effect transistor is a voltage operated device. Whereas Bipolar junction transistor is a current controlled device. Unlike BJT a FET requires virtually no input current.
3. This gives it an extremely high input resistance , which is its most important advantage over a bipolar transistor.
4. FET is also a three terminal device, labeled as source, drain and gate.
5. The source can be viewed as BJT's emitter, the drain as collector, and the gate as the counter part of the base.
6. The material that connects the source to drain is referred to as the channel.
7. FET operation depends only on the flow of majority carriers ,therefore they are called uni polar devices. BJT operation depends on both minority and majority carriers.
8. As FET has conduction through only majority carriers it is less noisy than BJT.
9. FETs are much easier to fabricate and are particularly suitable for ICs because they occupy less space than BJTs.
10. FET amplifiers have low gain bandwidth product due to the junction capacitive effects and produce more signal distortion except for small signal operation.
11. The performance of FET is relatively unaffected by ambient temperature changes. As it has a negative temperature coefficient at high current levels, it prevents the FET from thermal breakdown. The BJT has a positive temperature coefficient at high current levels which leads to thermal breakdown.

CLASSIFICATION OF FET:

There are two major categories of field effect transistors:

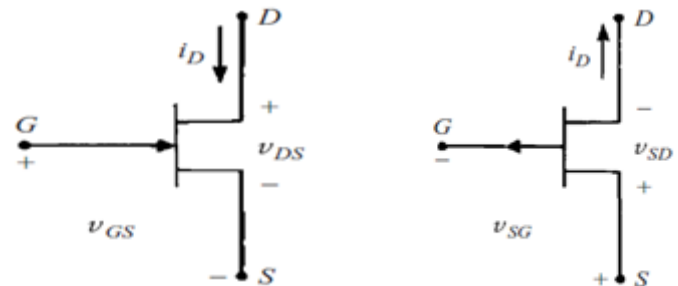
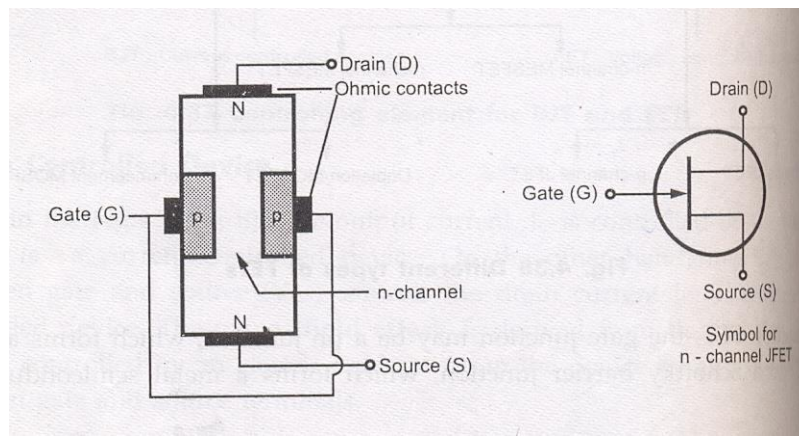
1. Junction Field Effect Transistors
2. MOSFETs

1. Junction Field Effect Transistors

- Junction Field Effect Transistors are further sub divided in to P- channel and N-channel devices.
- When the channel is of N-type the JFET is referred to as an N-channel JFET ,when the channel is of P-type the JFET is referred to as P-channel JFET.

CONSTRUCTION OF N-CHANNEL JFET

A piece of N- type material, referred to as channel has two smaller pieces of P-type material attached to its sides, forming PN junctions. The channel ends are designated as the drain and source . And the two pieces of P-type material are connected together and their terminal is called the gate. Since this channel is in the N-type bar, the FET is known as N-channel JFET.



N-channel FET P-channel FET

The schematic symbols for the P-channel and N-channel JFETs are shown in the figure

OPERATION OF N-CHANNEL JFET:-

The overall operation of the JFET is based on varying the width of the channel to control the drain current.

A piece of N type material referred to as the channel, has two smaller pieces of P type material attached to its sites, forming PN –Junctions. The channel’s ends are designated the drain and the source. And the two pieces of P type material are connected together and their terminal is called the gate. With the gate terminal not connected and the potential applied positive at the drain negative at the source a drain current I_D flows.

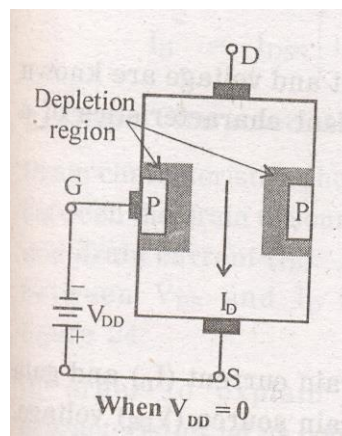
When the gate is biased negative with respect to the source the PN junctions are reverse biased and depletion regions are formed. The channel is more lightly doped than the P type gate blocks, so the depletion regions penetrate deeply into the channel. Since depletion region is a region depleted of charge carriers it behaves as an Insulator. The result is that the channel is narrowed. Its resistance is increased and I_D is reduced. When the negative gate bias voltage is further increased, the depletion regions meet at the center and I_D is cut off completely.

There are two ways to control the channel width

1. By varying the value of V_{GS}
2. And by Varying the value of V_{DS} holding V_{GS} constant

1 By varying the value of V_{GS} :-

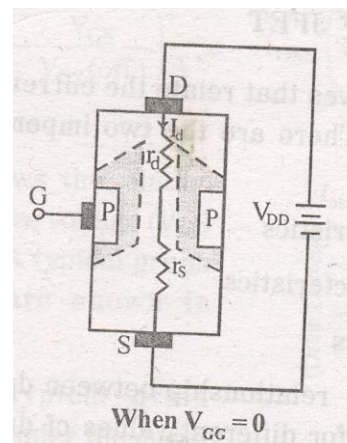
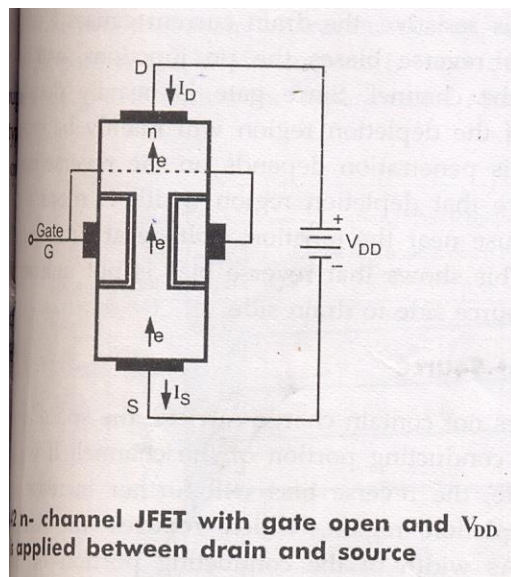
We can vary the width of the channel and in turn vary the amount of drain current. This can be done by varying the value of V_{GS} . This point is illustrated in the fig below. Here we are dealing with N channel FET. So channel is of N type and gate is of P type that constitutes a PN junction. This PN junction is always reverse biased in JFET operation .The reverse bias is applied by a battery voltage V_{GS} connected between the gate and the source terminal i.e positive terminal of the battery is connected to the source and negative terminal to gate.



- 1) When a PN junction is reverse biased the electrons and holes diffuse across junction by leaving immobile ions on the N and P sides , the region containing these immobile ions is known as depletion regions.
- 2) If both P and N regions are heavily doped then the depletion region extends symmetrically on both sides.
- 3) But in N channel FET P region is heavily doped than N type thus depletion region extends more in N region than P region.
- 4) So when no V_{DS} is applied the depletion region is symmetrical and the conductivity becomes Zero. Since there are no mobile carriers in the junction.
- 5) As the reverse bias voltage is increases the thickness of the depletion region also increases. i.e. the effective channel width decreases .
- 6) By varying the value of V_{GS} we can vary the width of the channel.

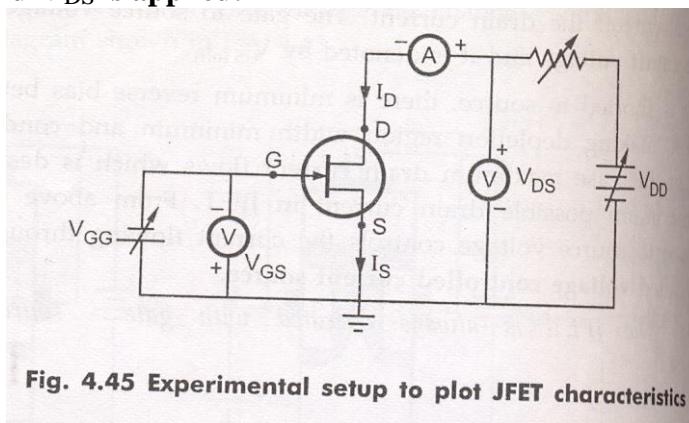
2 Varying the value of V_{DS} holding V_{GS} constant :-

- 1) When no voltage is applied to the gate i.e. $V_{GS} = 0$, V_{DS} is applied between source and drain the electrons will flow from source to drain through the channel constituting drain current I_D .
- 2) With $V_{GS} = 0$ for $I_D = 0$ the channel between the gate junctions is entirely open .In response to a small applied voltage V_{DS} , the entire bar acts as a simple semi conductor resistor and the current I_D increases linearly with V_{DS} .
- 3) The channel resistances are represented as R_D and R_S as shown in the fig.



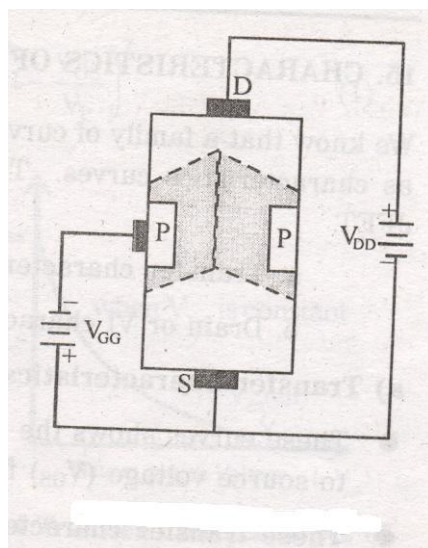
- 4) This increasing drain current I_D produces a voltage drop across r_d which reverse biases the gate to source junction, ($R_D > R_S$). Thus the depletion region is formed which is not symmetrical.
- 5) The depletion region i.e. developed penetrates deeper in to the channel near drain and less towards source because $V_{R_D} \gg V_{R_S}$. So reverse bias is higher near drain than at source.
- 6) As a result growing depletion region reduces the effective width of the channel. Eventually a voltage V_{DS} is reached at which the channel is pinched off. This is the voltage where the current I_D begins to level off and approach a constant value.
- 7) So, by varying the value of V_{DS} we can vary the width of the channel holding V_{GS} constant.

When both V_{GS} and V_{DS} is applied:-



It is of course in principle not possible for the channel to close Completely and there by reduce the current I_D to Zero for, if such indeed, could be the case the gate voltage V_{GS} is applied in the direction to provide additional reverse bias

- 1) When voltage is applied between the drain and source with a battery V_{DD} , the electrons flow from source to drain through the narrow channel existing between the depletion regions. This constitutes the drain current I_D , its conventional direction is from drain to source.
- 2) The value of drain current is maximum when no external voltage is applied between gate and source and is designated by I_{DSS} .



- 3) When V_{GS} is increased beyond Zero the depletion regions are widened. This reduces the effective width of the channel and therefore controls the flow of drain current through the channel.
- 4) When V_{GS} is further increased a stage is reached at which the depletion regions touch each other that means the entire channel is closed with depletion region. This reduces the drain current to Zero.

CHARACTERISTICS OF N-CHANNEL JFET :-

The family of curves that shows the relation between current and voltage are known as characteristic curves.

There are two important characteristics of a JFET.

- 1) Drain or VI Characteristics
- 2) Transfer characteristics

1. Drain Characteristics:

Drain characteristics shows the relation between the drain to source voltage V_{DS} and drain current I_D . In order to explain typical drain characteristics let us consider the curve with $V_{GS} = 0\text{ V}$.

- 1) When V_{DS} is applied and it is increasing the drain current I_D also increases linearly up to knee point.
- 2) This shows that FET behaves like an ordinary resistor. This region is called as ohmic region.
- 3) I_D increases with increase in drain to source voltage. Here the drain current is increased slowly as compared to ohmic region.

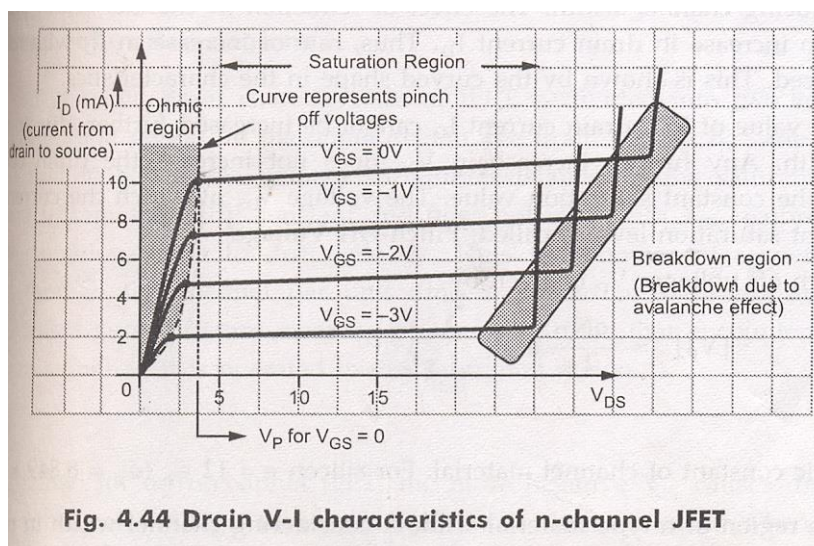


Fig. 4.44 Drain V-I characteristics of n-channel JFET

- 4) It is because of the fact that there is an increase in V_{DS} . This in turn increases the reverse bias voltage across the gate source junction. As a result of this depletion region grows in size thereby reducing the effective width of the channel.
- 5) All the drain to source voltage corresponding to point the channel width is reduced to a minimum value and is known as pinch off.
- 6) The drain to source voltage at which channel pinch off occurs is called pinch off voltage (V_P).

PINCH OFF Region:

- 1) This is the region shown by the curve as saturation region.
- 2) It is also called as saturation region or constant current region. Because of the channel is occupied with depletion region, the depletion region is more towards the drain and less towards the source, so the channel is limited, with this only limited number of carriers are only allowed to cross this channel from source drain causing a current that is constant in this region. To use FET as an amplifier it is operated in this saturation region.
- 3) In this drain current remains constant at its maximum value I_{DSS} .
- 4) The drain current in the pinch off region depends upon the gate to source voltage and is given by the relation

$$I_D = I_{DSS} [1 - V_{GS} / V_P]^2$$

This is known as shokley's relation.

BREAKDOWN REGION:

- 1) The region is shown by the curve. In this region, the drain current increases rapidly as the drain to source voltage is increased.
- 2) It is because of the gate to source junction due to avalanche effect.
- 3) The avalanche break down occurs at progressively lower value of V_{DS} because the reverse bias gate voltage adds to the drain voltage thereby increasing effective voltage across the gate junction

This causes

1. The maximum saturation drain current is smaller
2. The ohmic region portion decreased.
- 4) It is important to note that the maximum voltage V_{DS} which can be applied to FET is the lowest voltage which causes available break down.

2. TRANSFER CHARACTERISTICS:

These curves shows the relationship between drain current I_D and gate to source voltage V_{GS} for different values of V_{DS}

- 1) First adjust the drain to source voltage to some suitable value, then increase the gate to source voltage in small suitable value.
- 2) Plot the graph between gate to source voltage along the horizontal axis and current I_D on the vertical axis. We shall obtain a curve like this.

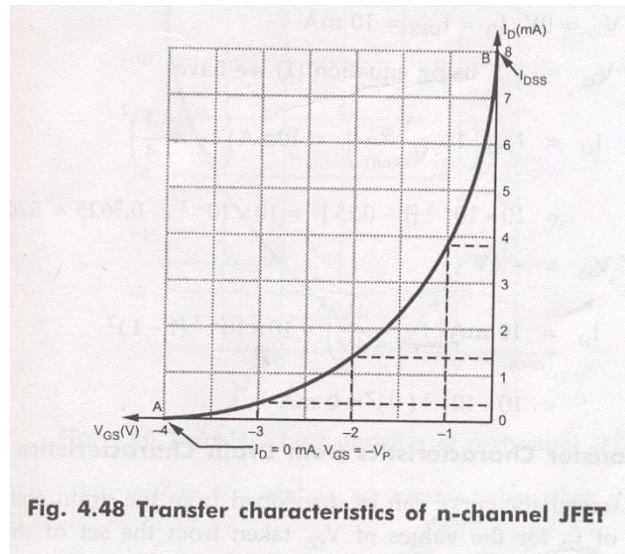


Fig. 4.48 Transfer characteristics of n-channel JFET

- 3) As we know that if V_{GS} is more negative curves drain current to reduce . where V_{GS} is made sufficiently negative, I_D is reduced to zero. This is caused by the widening of the depletion region to a point where it is completely closes the channel. The value of V_{GS} at the cutoff point is designed as $V_{GS\ off}$
- 4) The upper end of the curve as shown by the drain current value is equal to I_{DSS} that is when $V_{GS} = 0$ the drain current is maximum.
- 5) While the lower end is indicated by a voltage equal to $V_{GS\ off}$
- 6) If V_{GS} continuously increasing , the channel width is reduced , then $I_D = 0$
- 7) It may be noted that curve is part of the parabola; it may be expressed as

$$I_D = I_{DSS} [1 - V_{GS} / V_{GS\ off}]^2$$

DIFFERENCE BETWEEN V_P AND $V_{GS\ off}$:

- 1) V_P is the value of V_{GS} that causes the JFET to become constant current component, It is measured at $V_{GS} = 0V$ and has a constant drain current of $I_D = I_{DSS}$.
 .Where $V_{GS\ off}$ is the value of V_{GS} that reduces I_D to approximately zero.

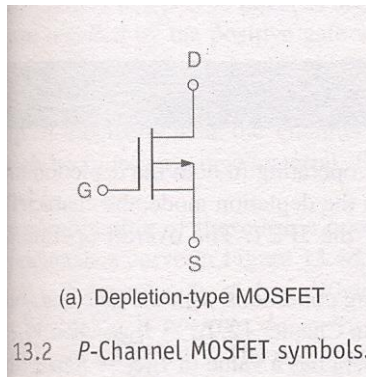
Why the gate to source junction of a JFET be always reverse biased ?

The gate to source junction of a JFET is never allowed to become forward biased because the gate material is not designed to handle any significant amount of current. If the junction is allowed to become forward biased, current is generated through the gate material. This current may destroy the component.

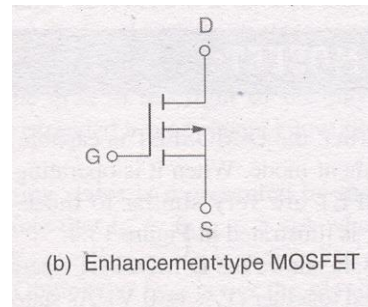
There is one more important characteristic of JFET reverse biasing i.e. J FET 's have extremely high characteristic gate input impedance. This impedance is typically in the high mega ohm range. With the advantage of extremely high input impedance it draws no current from the source. The high input impedance of the JFET has led to its extensive use in integrated circuits. The low current requirements of the component makes it perfect for use in ICs. Where thousands of transistors must be etched on to a single piece of silicon. The low current draw helps the IC to remain relatively cool, thus allowing more components to be placed in a smaller physical area.

MOSFET:

- We now turn our attention to the Insulated Gate FET or Metal Oxide Semi Conductor FET which is having the greater commercial importance than the junction FET.
- Most MOSFETS however are triodes, with the substrate internally connected to the source. The circuit symbols used by several manufacturers are indicated in the Fig below.



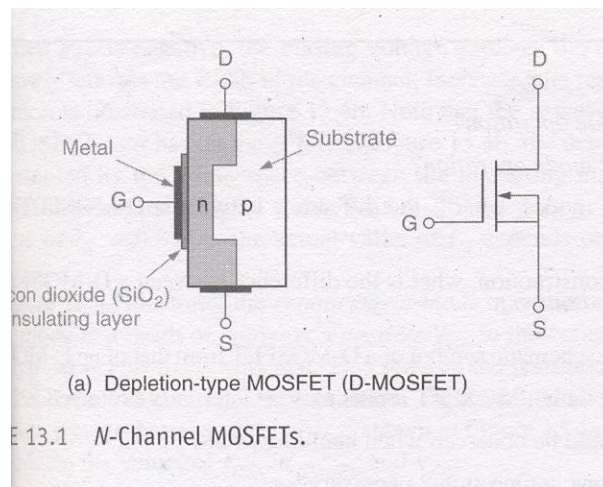
(a) Depletion type MOSFET



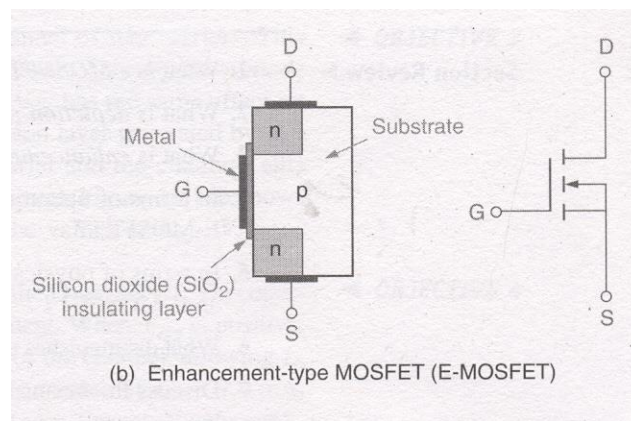
(b) Enhancement type MOSFET

Both of them are P- channel

- Here are two basic types of MOSFETS
 - (1) Depletion type
 - (2) Enhancement type MOSFET.
- D-MOSFETS can be operated in both the depletion mode and the enhancement mode.
- E MOSFETS are restricted to operate in enhancement mode. The primary difference between them is their physical construction.
- The construction difference between the two is shown in the fig given below.



As we can see the D MOSFET have physical channel between the source and drain terminals(Shaded area)



The E MOSFET on the other hand has no such channel physically. It depends on the gate voltage to form a channel between the source and the drain terminals.

Both MOSFETS have an insulating layer between the gate and the rest of the component. This insulating layer is made up of SiO_2 a glass like insulating material. The gate material is made up of metal conductor. Thus going from gate to substrate, we can have metal oxide semiconductor which is where the term MOSFET comes from.

Since the gate is insulated from the rest of the component, the MOSFET is sometimes referred to as an insulated gate FET or IGFET.

The foundation of the MOSFET is called the substrate. This material is represented in the schematic symbol by the center line that is connected to the source.

In the symbol for the MOSFET, the arrow is placed on the substrate. As with JFET an arrow pointing in represents an N-channel device, while an arrow pointing out represents p-channel device.

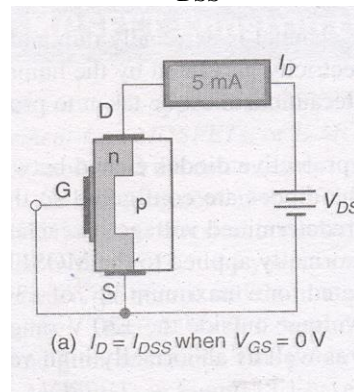
CONSTRUCTION OF AN N-CHANNEL MOSFET :

The N- channel MOSFET consists of a lightly doped p type substance into which two heavily doped n+ regions are diffused as shown in the Fig. These n+ sections , which will act as source and drain.

A thin layer of insulation silicon dioxide (SiO_2) is grown over the surface of the structure, and holes are cut into oxide layer, allowing contact with the source and drain. Then the gate metal area is overlaid on the oxide, covering the entire channel region. Metal contacts are made to drain and source and the contact to the metal over the channel area is the gate terminal. The metal area of the gate, in conjunction with the insulating dielectric oxide layer and the semiconductor channel, forms a parallel plate capacitor. The insulating layer of SiO_2 is the reason why this device is called the insulated gate field effect transistor. This layer results in an extremely high input resistance (10^{10} to 10^{15} ohms) for MOSFET.

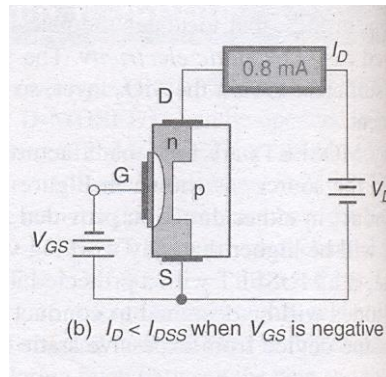
DEPLETION MOSFET

The basic structure of D –MOSFET is shown in the fig. An N-channel is diffused between source and drain with the device an appreciable drain current I_{DSS} flows for zero gate to source voltage, $V_{GS}=0$.



Depletion mode operation:-

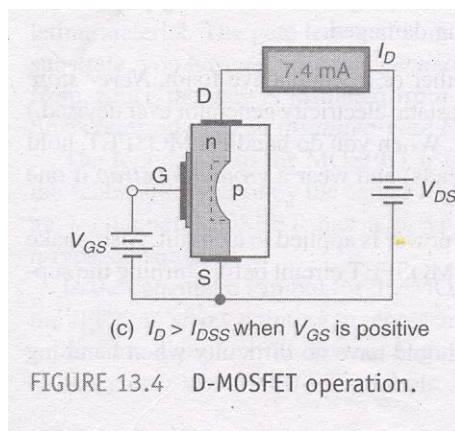
- 1) The above fig shows the D-MOSFET operating conditions with gate and source terminals shorted together ($V_{GS}=0V$)
- 2) At this stage $I_D = I_{DSS}$ where $V_{GS} = 0V$, with this voltage V_{DS} , an appreciable drain current I_{DSS} flows.
- 3) If the gate to source voltage is made negative i.e. V_{GS} is negative. Positive charges are induced in the channel through the SiO_2 of the gate capacitor.
- 4) Since the current in a FET is due to majority carriers (electrons for an N-type material), the induced positive charges make the channel less conductive and the drain current drops as V_{GS} is made more negative.
- 5) The re distribution of charge in the channel causes an effective depletion of majority carriers, which accounts for the designation depletion MOSFET.
- 6) That means biasing voltage V_{GS} depletes the channel of free carriers This effectively reduces the width of the channel, increasing its resistance.
- 7) Note that negative V_{GS} has the same effect on the MOSFET as it has on the JFET.



- 8) As shown in the fig above, the depletion layer generated by V_{GS} (represented by the white space between the insulating material and the channel) cuts into the channel, reducing its width. As a result, $I_D < I_{DSS}$. The actual value of I_D depends on the value of I_{DSS} , $V_{GS(OFF)}$ and V_{GS}

Enhancement mode operation of the D-MOSFET:-

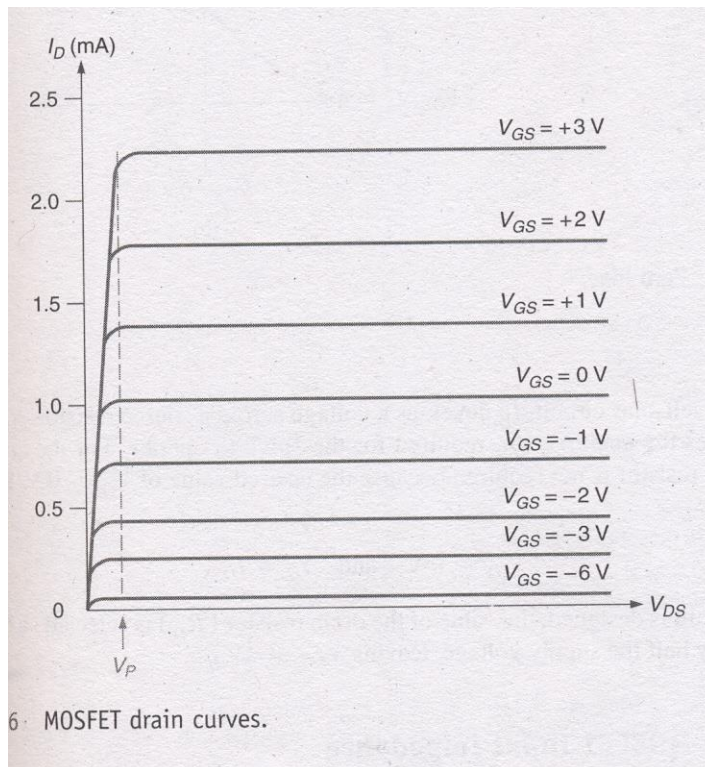
- 1) This operating mode is a result of applying a positive gate to source voltage V_{GS} to the device.
- 2) When V_{GS} is positive the channel is effectively widened. This reduces the resistance of the channel allowing I_D to exceed the value of I_{DSS}
- 3) When V_{GS} is given positive the majority carriers in the p-type are holes. The holes in the P type substrate are repelled by the +ve gate voltage.
- 4) At the same time, the conduction band electrons (minority carriers) in the P type material are attracted towards the channel by the +gate voltage.
- 5) With the build up of electrons near the channel, the area to the right of the physical channel effectively becomes an N type material.
- 6) The extended n type channel now allows more current, $I_D > I_{DSS}$



Characteristics of Depletion MOSFET:-

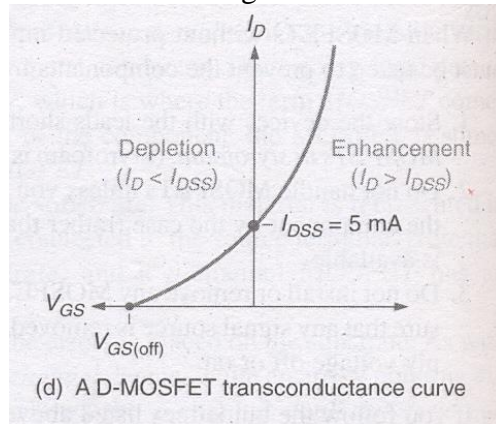
The fig. shows the drain characteristics for the N channel depletion type MOSFET

- 1) The curves are plotted for both V_{GS} positive and V_{GS} negative voltages
- 2) When $V_{GS} = 0$ and negative the MOSFET operates in depletion mode when V_{GS} is positive, the MOSFET operates in the enhancement mode.
- 3) The difference between JFET and D MOSFET is that JFET does not operate for positive values of V_{GS} .
- 4) When $V_{DS} = 0$, there is no conduction takes place between source to drain, if $V_{GS} < 0$ and $V_{DS} > 0$ then I_D increases linearly.
- 5) But as $V_{GS} = 0$ induces positive charges holes in the channel, and controls the channel width. Thus the conduction between source to drain is maintained as constant, i.e. I_D is constant.
- 6) If $V_{GS} > 0$ the gate induces more electrons in channel side, it is added with the free electrons generated by source. again the potential applied to gate determines the channel width and maintains constant current flow through it as shown in Fig



TRANSFER CHARACTERISTICS:-

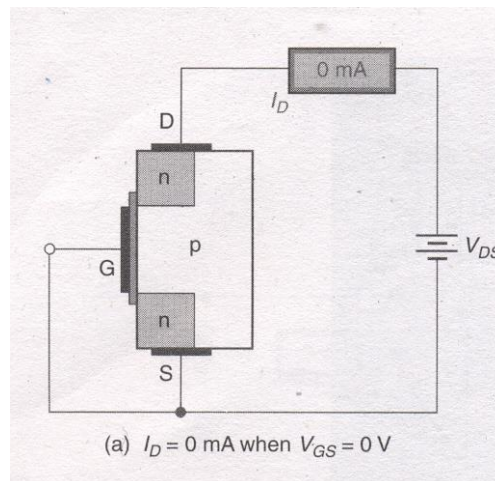
The combination of 3 operating states i.e. $V_{GS} = 0V$, $V_{GS} < 0V$, $V_{GS} > 0V$ is represented by the D MOSFET transconductance curve shown in Fig.



- 1) Here in this curve it may be noted that the region AB of the characteristics similar to that of JFET.
- 2) This curve extends for the positive values of V_{GS}
- 3) Note that $I_D = I_{DSS}$ for $V_{GS} = 0V$ when V_{GS} is negative, $I_D < I_{DSS}$ when $V_{GS} = V_{GS}(\text{off})$, I_D is reduced to approximately 0mA. Where V_{GS} is positive $I_D > I_{DSS}$. So obviously I_{DSS} is not the maximum possible value of I_D for a MOSFET.
- 4) The curves are similar to JFET so that the D MOSFET have the same transconductance equation.

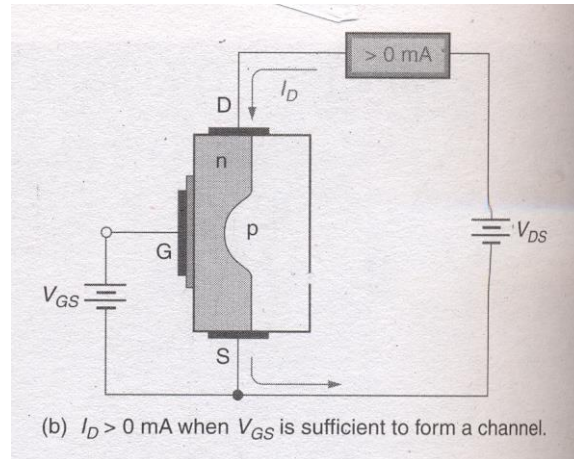
E-MOSFETS

The E MOSFET is capable of operating only in the enhancement mode. The gate potential must be positive w.r.t to source.



- 1) when the value of $V_{GS} = 0V$, there is no channel connecting the source and drain materials.
- 2) As a result, there can be no significant amount of drain current.
- 3) When $V_{GS} = 0$, the V_{DD} supply tries to force free electrons from source to drain but the presence of p-region does not permit the electrons to pass through it. Thus there is no drain current at $V_{GS} = 0$,
- 4) If V_{GS} is positive, it induces a negative charge in the p type substrate just adjacent to the SiO_2 layer.

- 5) As the holes are repelled by the positive gate voltage, the minority carrier electrons attracted toward this voltage. This forms an effective N type bridge between source and drain providing a path for drain current.
- 6) This +ve gate voltage forma a channel between the source and drain.
- 7) This produces a thin layer of N type channel in the P type substarate.This layer of free electrons is called N type inversion layer.

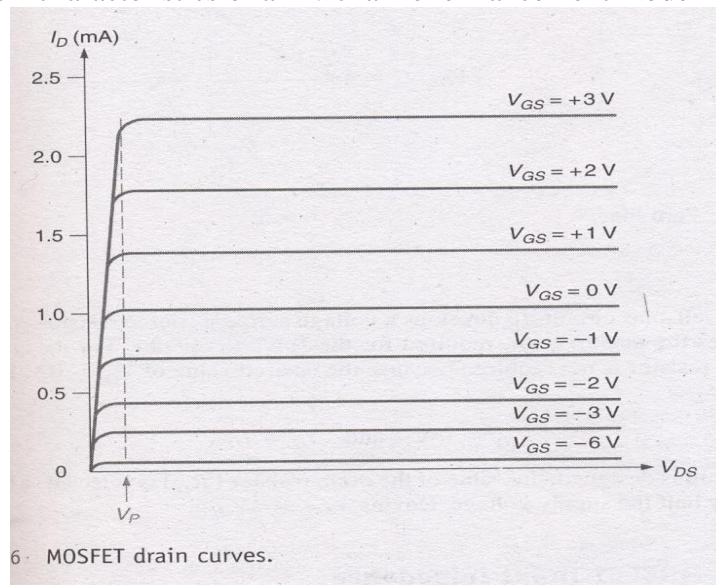


- 8) The minimum V_{GS} which produces this inversion layer is called threshold voltage and is designated by $V_{GS(th)}$. This is the point at which the device turns on is called the threshold voltage $V_{GS(th)}$
- 9) When the voltage V_{GS} is $< V_{GS(th)}$ no current flows from drain to source.
- 10) How ever when the voltage $V_{GS} > V_{GS(th)}$ the inversion layer connects the drain to source and we get significant values of current.

CHARACTERISTICS OF E MOSFET:

1. DRAIN CHARACTERISTICS:

The volt ampere drain characteristics of an N-channel enhancement mode MOSFET are given in.



2. TRANSFER CHARACTERISTICS:

- 1) The current I_{DSS} at $V_{GS} \leq 0$ is very small being of the order of a few nano amps.
- 2) As V_{GS} is made +ve, the current I_D increases slowly at first, and then much more rapidly with an increase in V_{GS} .
- 3) The standard transconductance formula will not work for the E MOSFET.
- 4) To determine the value of I_D at a given value of V_{GS} we must use the following relation

$$I_D = K[V_{GS} - V_{GS(th)}]^2$$

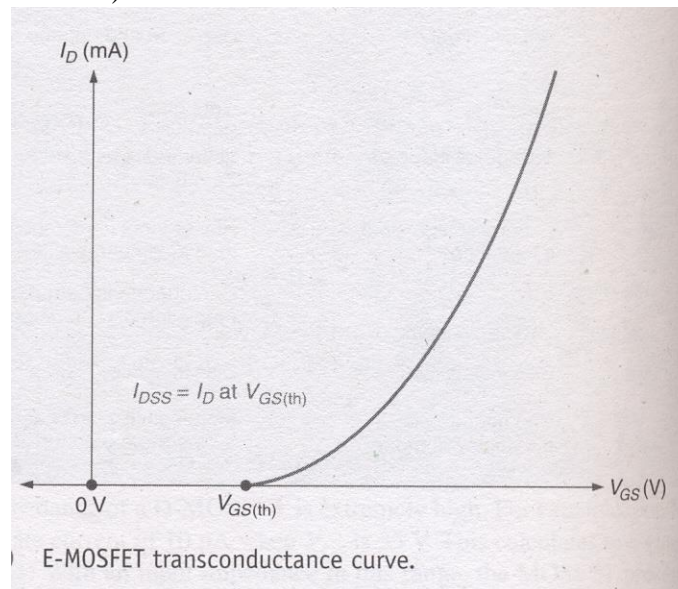
Where K is constant for the MOSFET. found as

$$K = \frac{I_{D(on)}}{[v_{gs(on)} - v_{gs(th)}]^2}$$

From the data specification sheets, the 2N7000 has the following ratings.

$I_{D(on)} = 75\text{mA}$ (minimum).

And $V_{GS(th)} = 0.8$ (minimum)



APPLICATION OF MOSFET

One of the primary contributions to electronics made by MOSFETs can be found in the area of digital (computer electronics). The signals in digital circuits are made up of rapidly switching dc levels. This signal is called as a rectangular wave, made up of two dc levels (or logic levels). These logic levels are 0V and +5V.

A group of circuits with similar circuitry and operating characteristics is referred to as a logic family. All the circuits in a given logic family respond to the same logic levels, have similar speed and power-handling capabilities, and can be directly connected together. One such logic family is complementary MOS (or CMOS) logic. This logic family is made up entirely of MOSFETs.

COMPARISON OF MOSFET WITH JFET

- a. In enhancement and depletion types of MOSFET, the transverse electric field induced across an insulating layer deposited on the semiconductor material controls the conductivity of the channel.
- b. In the JFET the transverse electric field across the reverse biased PN junction controls the conductivity of the channel.
- c. The gate leakage current in a MOSFET is of the order of 10^{-12}A . Hence the input resistance of a MOSFET is very high in the order of 10^{10} to $10^{15}\ \Omega$. The gate leakage current of a JFET is of the order of 10^{-9}A , and its input resistance is of the order of $10^8\ \Omega$.

- d. The output characteristics of the JFET are flatter than those of the MOSFET, and hence the drain resistance of a JFET (0.1 to 1M Ω) is much higher than that of a MOSFET (1 to 50k Ω).
- e. JFETs are operated only in the depletion mode. The depletion type MOSFET may be operated in both depletion and enhancement mode.
- f. Comparing to JFET, MOSFETs are easier to fabricate.
- g. Special digital CMOS circuits are available which involve near zero power dissipation and very low voltage and current requirements. This makes them suitable for portable systems.